4 | DIFFRACTION

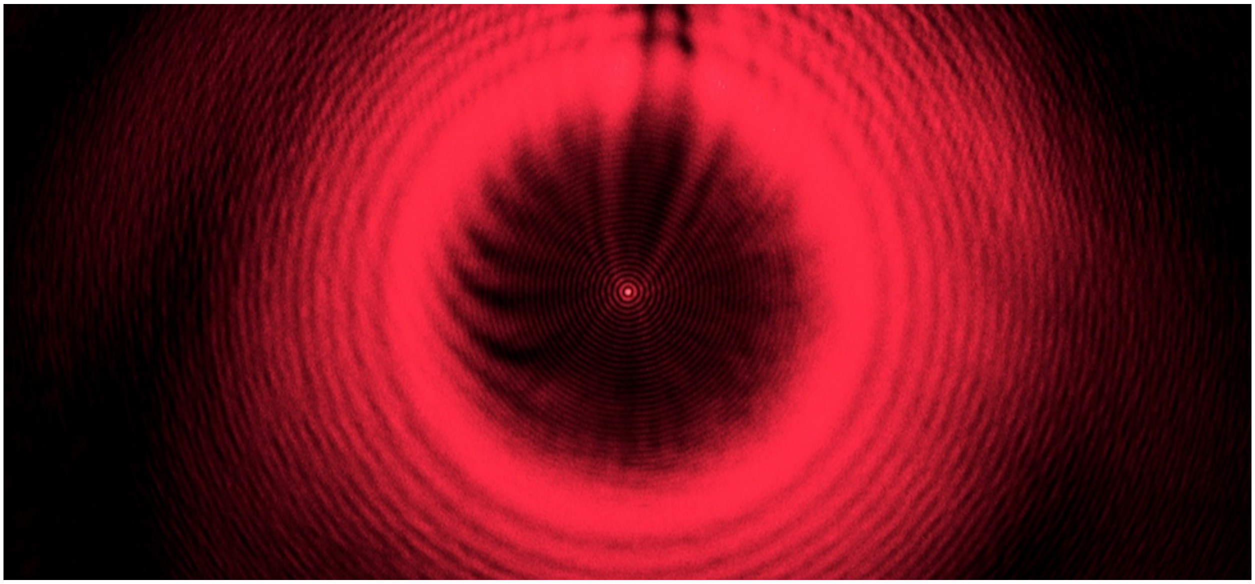


Figure 4.1 A steel ball bearing illuminated by a laser does not cast a sharp, circular shadow. Instead, a series of diffraction fringes and a central bright spot are observed. Known as Poisson's spot, the effect was first predicted by Augustin-Jean Fresnel (1788–1827) as a consequence of diffraction of light waves. Based on principles of ray optics, Siméon-Denis Poisson (1781–1840) argued against Fresnel's prediction. (credit: modification of work by Harvard Natural Science Lecture Demonstrations)

Chapter Outline

Introduction

Imagine passing a monochromatic light beam through a narrow opening—a slit just a little wider than the wavelength of the light. Instead of a simple shadow of the slit on the screen, you will see that an interference pattern appears, even though there is only one slit.

In the chapter on interference, we saw that you need two sources of waves for interference to occur. How can there be an interference pattern when we have only one slit? In **The Nature of Light**, we learned that, due to Huygens's principle, we can imagine a wave front as equivalent to infinitely many point sources of waves. Thus, a wave from a slit can behave not as one wave but as an infinite number of point sources. These waves can interfere with each other, resulting in an interference pattern without the presence of a second slit. This phenomenon is called *diffraction*.

Another way to view this is to recognize that a slit has a small but finite width. In the preceding chapter, we implicitly regarded slits as objects with positions but no size. The widths of the slits were considered negligible. When the slits have finite widths, each point along the opening can be considered a point source of light—a foundation of Huygens's principle. Because real-world optical instruments must have finite apertures (otherwise, no light can enter), diffraction plays a major role in the way we interpret the output of these optical instruments. For example, diffraction places limits on our ability to resolve images or objects. This is a problem that we will study later in this chapter.

4.1 | Single-Slit Diffraction

Learning Objectives

By the end of this section, you will be able to:

- Explain the phenomenon of diffraction and the conditions under which it is observed
- Describe diffraction through a single slit

After passing through a narrow aperture (opening), a wave propagating in a specific direction tends to spread out. For example, sound waves that enter a room through an open door can be heard even if the listener is in a part of the room where the geometry of ray propagation dictates that there should only be silence. Similarly, ocean waves passing through an opening in a breakwater can spread throughout the bay inside. (Figure 4.2). The spreading and bending of sound and ocean waves are two examples of **diffraction**, which is the bending of a wave around the edges of an opening or an obstacle—a phenomenon exhibited by all types of waves.

Figure 4.2 Because of the diffraction of waves, ocean waves entering through an opening in a breakwater can spread throughout the bay. (credit: modification of map data from Google Earth)

The diffraction of sound waves is apparent to us because wavelengths in the audible region are approximately the same size as the objects they encounter, a condition that must be satisfied if diffraction effects are to be observed easily. Since the wavelengths of visible light range from approximately 390 to 770 nm, most objects do not diffract light significantly. However, situations do occur in which apertures are small enough that the diffraction of light is observable. For example, if you place your middle and index fingers close together and look through the opening at a light bulb, you can see a rather clear diffraction pattern, consisting of light and dark lines running parallel to your fingers.

Diffraction through a Single Slit

Light passing through a single slit forms a diffraction pattern somewhat different from those formed by double slits or diffraction gratings, which we discussed in the chapter on interference. Figure 4.3 shows a single-slit diffraction pattern. Note that the central maximum is larger than maxima on either side and that the intensity decreases rapidly on either side. In contrast, a diffraction grating (Diffraction Gratings) produces evenly spaced lines that dim slowly on either side of the center.

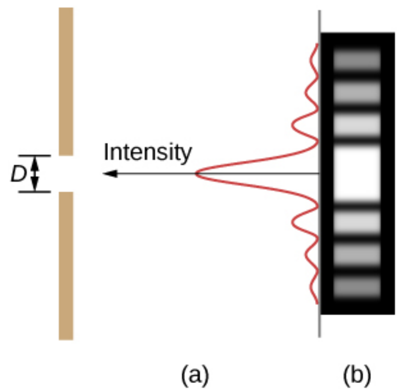


Figure 4.3 Single-slit diffraction pattern. (a) Monochromatic light passing through a single slit has a central maximum and many smaller and dimmer maxima on either side. The central maximum is six times higher than shown. (b) The diagram shows the bright central maximum, and the dimmer and thinner maxima on either side.

The analysis of single-slit diffraction is illustrated in **Figure 4.4**. Here, the light arrives at the slit, illuminating it uniformly and is in phase across its width. We then consider light propagating onwards from different parts of the *same* slit. According to Huygens's principle, every part of the wave front in the slit emits wavelets, as we discussed in **The Nature of Light**. These are like rays that start out in phase and head in all directions. (Each ray is perpendicular to the wave front of a wavelet.) Assuming the screen is very far away compared with the size of the slit, rays heading toward a common destination are nearly parallel. When they travel straight ahead, as in part (a) of the figure, they remain in phase, and we observe a central maximum. However, when rays travel at an angle θ relative to the original direction of the beam, each ray travels a different distance to a common location, and they can arrive in or out of phase. In part (b), the ray from the bottom travels a distance of one wavelength λ farther than the ray from the top. Thus, a ray from the center travels a distance $\lambda/2$ less than the one at the bottom edge of the slit, arrives out of phase, and interferes destructively. A ray from slightly above the center and one from slightly above the bottom also cancel one another. In fact, each ray from the slit interferes destructively with another ray. In other words, a pair-wise cancellation of all rays results in a dark minimum in intensity at this angle. By symmetry, another minimum occurs at the same angle to the right of the incident direction (toward the bottom of the figure) of the light.

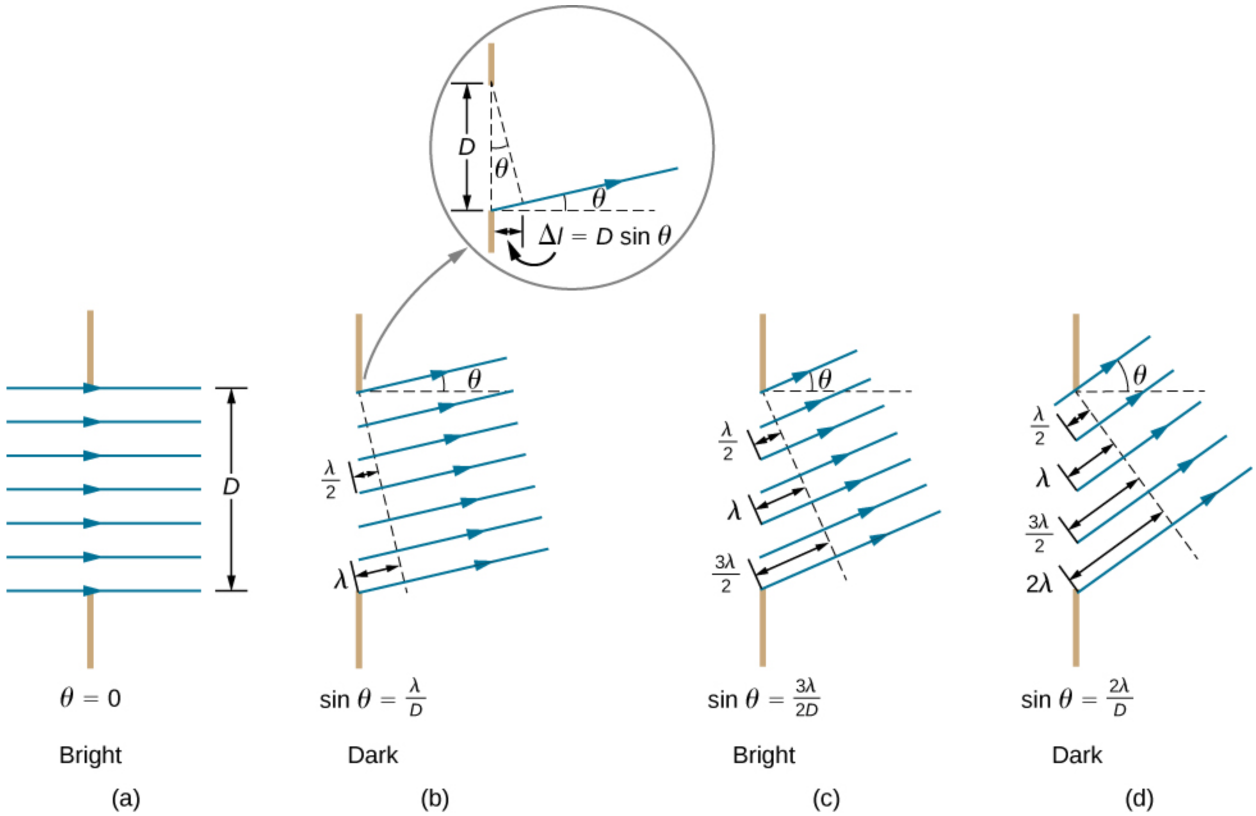


Figure 4.4 Light passing through a single slit is diffracted in all directions and may interfere constructively or destructively, depending on the angle. The difference in path length for rays from either side of the slit is seen to be $D \sin \theta$.

At the larger angle shown in part (c), the path lengths differ by $3\lambda/2$ for rays from the top and bottom of the slit. One ray travels a distance λ different from the ray from the bottom and arrives in phase, interfering constructively. Two rays, each from slightly above those two, also add constructively. Most rays from the slit have another ray to interfere with constructively, and a maximum in intensity occurs at this angle. However, not all rays interfere constructively for this situation, so the maximum is not as intense as the central maximum. Finally, in part (d), the angle shown is large enough to produce a second minimum. As seen in the figure, the difference in path length for rays from either side of the slit is $D \sin \theta$, and we see that a destructive minimum is obtained when this distance is an integral multiple of the wavelength.

Thus, to obtain **destructive interference for a single slit**,

$$D \sin \theta = m\lambda, \text{ for } m = \pm 1, \pm 2, \pm 3, \ldots \text{(destructive)}, \tag{4.1}$$

where D is the slit width, λ is the light's wavelength, θ is the angle relative to the original direction of the light, and m is the order of the minimum. **Figure 4.5** shows a graph of intensity for single-slit interference, and it is apparent that the maxima on either side of the central maximum are much less intense and not as wide. This effect is explored in **Double-Slit Diffraction**.

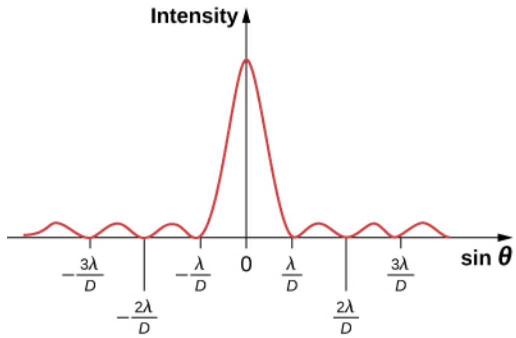


Figure 4.5 A graph of single-slit diffraction intensity showing the central maximum to be wider and much more intense than those to the sides. In fact, the central maximum is six times higher than shown here.

Example 4.1

Calculating Single-Slit Diffraction

Visible light of wavelength 550 nm falls on a single slit and produces its second diffraction minimum at an angle of $45.0°$ relative to the incident direction of the light, as in **Figure 4.6**. (a) What is the width of the slit? (b) At what angle is the first minimum produced?

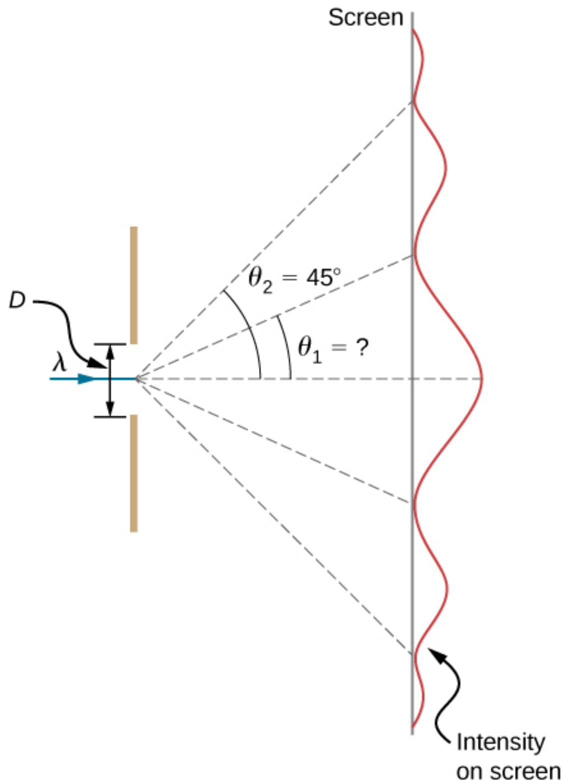


Figure 4.6 In this example, we analyze a graph of the single-slit diffraction pattern.

Strategy

From the given information, and assuming the screen is far away from the slit, we can use the equation $D \sin\theta = m\lambda$ first to find *D*, and again to find the angle for the first minimum θ_1.

Solution

a. We are given that $\lambda = 550 \text{ nm}$, $m = 2$, and $\theta_2 = 45.0°$. Solving the equation $D \sin\theta = m\lambda$ for D and substituting known values gives

$$D = \frac{m\lambda}{\sin\theta_2} = \frac{2(550 \text{ nm})}{\sin 45.0°} = \frac{1100 \times 10^{-9} \text{ m}}{0.707} = 1.56 \times 10^{-6} \text{ m}.$$

b. Solving the equation $D \sin\theta = m\lambda$ for $\sin\theta_1$ and substituting the known values gives

$$\sin\theta_1 = \frac{m\lambda}{D} = \frac{1(550 \times 10^{-9} \text{ m})}{1.56 \times 10^{-6} \text{ m}}.$$

Thus the angle θ_1 is

$$\theta_1 = \sin^{-1} 0.354 = 20.7°.$$

Significance

We see that the slit is narrow (it is only a few times greater than the wavelength of light). This is consistent with the fact that light must interact with an object comparable in size to its wavelength in order to exhibit significant wave effects such as this single-slit diffraction pattern. We also see that the central maximum extends $20.7°$ on either side of the original beam, for a width of about $41°$. The angle between the first and second minima is only about $24°$ $(45.0° - 20.7°)$. Thus, the second maximum is only about half as wide as the central maximum.

4.1 **Check Your Understanding** Suppose the slit width in **Example 4.1** is increased to 1.8×10^{-6} m. What are the new angular positions for the first, second, and third minima? Would a fourth minimum exist?

4.2 | Intensity in Single-Slit Diffraction

Learning Objectives

By the end of this section, you will be able to:

- Calculate the intensity relative to the central maximum of the single-slit diffraction peaks
- Calculate the intensity relative to the central maximum of an arbitrary point on the screen

To calculate the intensity of the diffraction pattern, we follow the phasor method used for calculations with ac circuits in **Alternating-Current Circuits (http://cnx.org/content/m58485/latest/)**. If we consider that there are N Huygens sources across the slit shown in **Figure 4.4**, with each source separated by a distance D/N from its adjacent neighbors, the path difference between waves from adjacent sources reaching the arbitrary point P on the screen is $(D/N) \sin\theta$. This distance is equivalent to a phase difference of $(2\pi D/\lambda N) \sin\theta$. The phasor diagram for the waves arriving at the point whose angular position is θ is shown in **Figure 4.7**. The amplitude of the phasor for each Huygens wavelet is ΔE_0, the amplitude of the resultant phasor is E, and the phase difference between the wavelets from the first and the last sources is

$$\phi = \left(\frac{2\pi}{\lambda}\right) D \sin\theta.$$

With $N \to \infty$, the phasor diagram approaches a circular arc of length $N\Delta E_0$ and radius r. Since the length of the arc is $N\Delta E_0$ for any ϕ, the radius r of the arc must decrease as ϕ increases (or equivalently, as the phasors form tighter spirals).

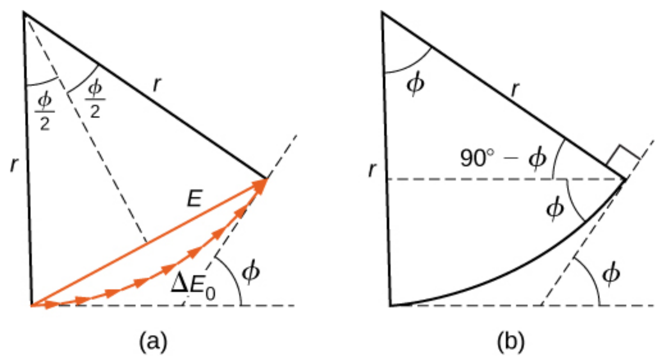


Figure 4.7 (a) Phasor diagram corresponding to the angular position θ in the single-slit diffraction pattern. The phase difference between the wavelets from the first and last sources is $\phi = (2\pi/\lambda)D\sin\theta$. (b) The geometry of the phasor diagram.

The phasor diagram for $\phi = 0$ (the center of the diffraction pattern) is shown in **Figure 4.8**(a) using $N = 30$. In this case, the phasors are laid end to end in a straight line of length $N\Delta E_0$, the radius *r* goes to infinity, and the resultant has its maximum value $E = N\Delta E_0$. The intensity of the light can be obtained using the relation $I = \frac{1}{2}c\varepsilon_0 E^2$ from **Electromagnetic Waves (http://cnx.org/content/m58495/latest/)** . The intensity of the maximum is then

$$I_0 = \frac{1}{2}c\varepsilon_0 (N\Delta E_0)^2 = \frac{1}{2\mu_0 c}(N\Delta E_0)^2,$$

where $\varepsilon_0 = 1/\mu_0 c^2$. The phasor diagrams for the first two zeros of the diffraction pattern are shown in parts (b) and (d) of the figure. In both cases, the phasors add to zero, after rotating through $\phi = 2\pi$ rad for $m = 1$ and 4π rad for $m = 2$.

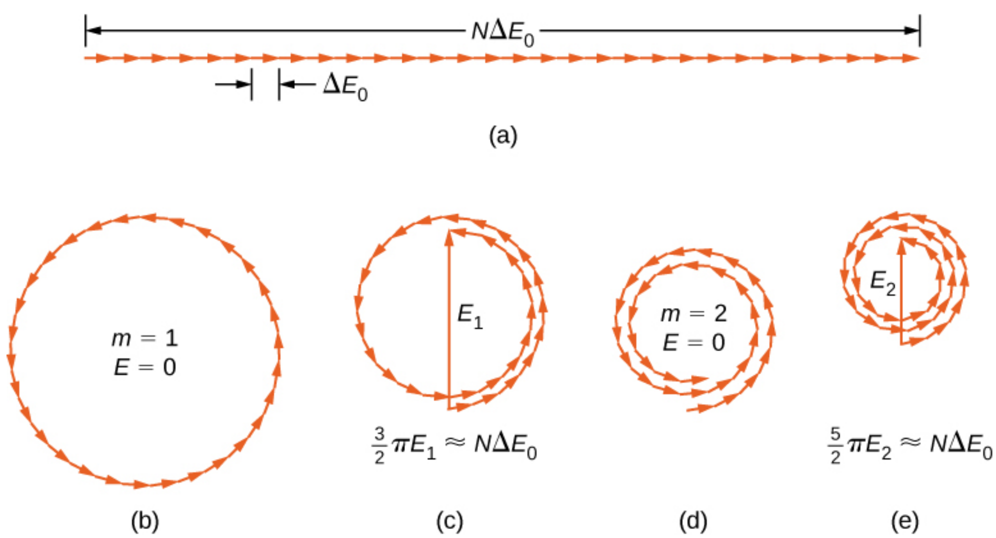


Figure 4.8 Phasor diagrams (with 30 phasors) for various points on the single-slit diffraction pattern. Multiple rotations around a given circle have been separated slightly so that the phasors can be seen. (a) Central maximum, (b) first minimum, (c) first maximum beyond central maximum, (d) second minimum, and (e) second maximum beyond central maximum.

The next two maxima beyond the central maxima are represented by the phasor diagrams of parts (c) and (e). In part (c), the phasors have rotated through $\phi = 3\pi$ rad and have formed a resultant phasor of magnitude E_1. The length of the arc formed by the phasors is $N\Delta E_0$. Since this corresponds to 1.5 rotations around a circle of diameter E_1, we have

$$\frac{3}{2}\pi E_1 \approx N\Delta E_0,$$

so

$$E_1 = \frac{2N\Delta E_0}{3\pi}$$

and

$$I_1 = \frac{1}{2\mu_0 c}E_1^2 = \frac{4(N\Delta E_0)^2}{(9\pi^2)(2\mu_0 c)} \approx 0.045 I_0,$$

where

$$I_0 = \frac{(N\Delta E_0)^2}{2\mu_0 c}.$$

In part (e), the phasors have rotated through $\phi = 5\pi$ rad, corresponding to 2.5 rotations around a circle of diameter E_2 and arc length $N\Delta E_0$. This results in $I_2 \approx 0.016 I_0$. The proof is left as an exercise for the student (**Exercise 4.119**).

These two maxima actually correspond to values of ϕ slightly less than 3π rad and 5π rad. Since the total length of the arc of the phasor diagram is always $N\Delta E_0$, the radius of the arc decreases as ϕ increases. As a result, E_1 and E_2 turn out to be slightly larger for arcs that have not quite curled through 3π rad and 5π rad, respectively. The exact values of ϕ for the maxima are investigated in **Exercise 4.120**. In solving that problem, you will find that they are less than, but very close to, $\phi = 3\pi, 5\pi, 7\pi, \ldots$ rad.

To calculate the intensity at an arbitrary point *P* on the screen, we return to the phasor diagram of **Figure 4.7**. Since the arc subtends an angle ϕ at the center of the circle,

$$N\Delta E_0 = r\phi$$

and

$$\sin\left(\frac{\phi}{2}\right) = \frac{E}{2r}.$$

where *E* is the amplitude of the resultant field. Solving the second equation for *E* and then substituting *r* from the first equation, we find

$$E = 2r\sin\frac{\phi}{2} = 2\frac{N\Delta E_o}{\phi}\sin\frac{\phi}{2}.$$

Now defining

$$\beta = \frac{\phi}{2} = \frac{\pi D \sin\theta}{\lambda} \quad (4.2)$$

we obtain

$$E = N\Delta E_0 \frac{\sin\beta}{\beta} \quad (4.3)$$

This equation relates the amplitude of the resultant field at any point in the diffraction pattern to the amplitude $N\Delta E_0$ at the central maximum. The intensity is proportional to the square of the amplitude, so

$$I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \tag{4.4}$$

where $I_0 = (N \Delta E_0)^2 / 2\mu_0 c$ is the intensity at the center of the pattern.

For the central maximum, $\phi = 0$, β is also zero and we see from l'Hôpital's rule that $\lim_{\beta \to 0} (\sin \beta / \beta) = 1$, so that $\lim_{\phi \to 0} I = I_0$. For the next maximum, $\phi = 3\pi$ rad, we have $\beta = 3\pi/2$ rad and when substituted into **Equation 4.4**, it yields

$$I_1 = I_0 \left(\frac{\sin 3\pi/2}{3\pi/2} \right)^2 \approx 0.045 I_0,$$

in agreement with what we found earlier in this section using the diameters and circumferences of phasor diagrams. Substituting $\phi = 5\pi$ rad into **Equation 4.4** yields a similar result for I_2.

A plot of **Equation 4.4** is shown in **Figure 4.9** and directly below it is a photograph of an actual diffraction pattern. Notice that the central peak is much brighter than the others, and that the zeros of the pattern are located at those points where $\sin \beta = 0$, which occurs when $\beta = m\pi$ rad. This corresponds to

$$\frac{\pi D \sin \theta}{\lambda} = m\pi,$$

or

$$D \sin \theta = m\lambda,$$

which is **Equation 4.1**.

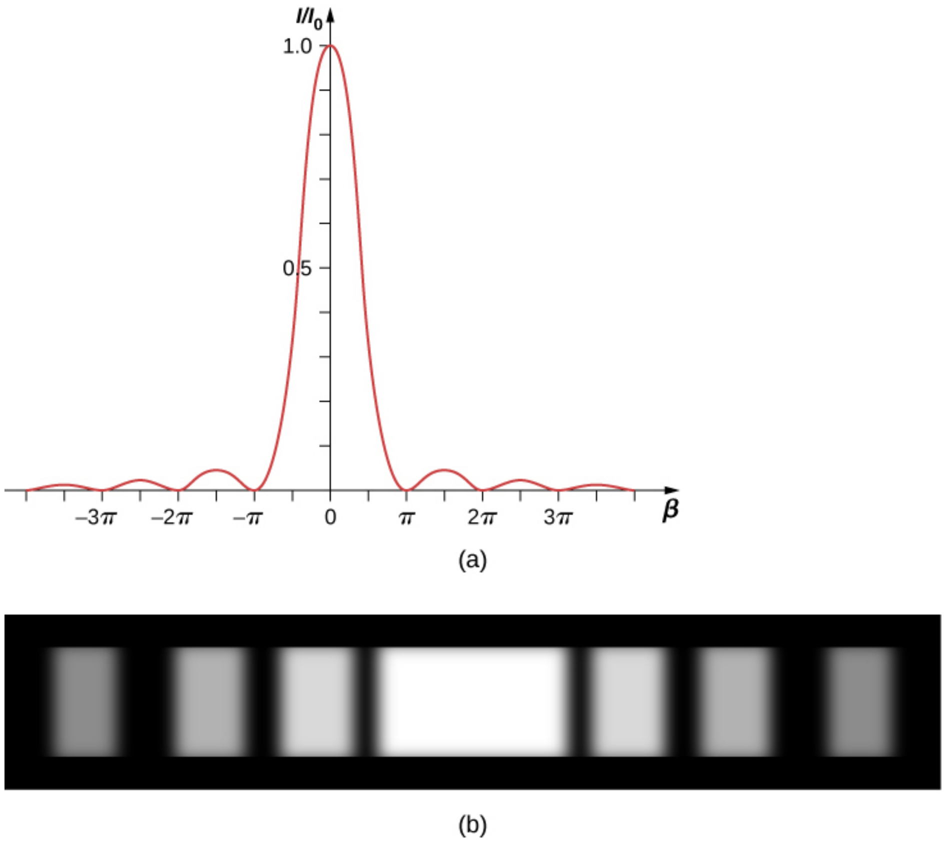


Figure 4.9 (a) The calculated intensity distribution of a single-slit diffraction pattern. (b) The actual diffraction pattern.

Example 4.2

Intensity in Single-Slit Diffraction

Light of wavelength 550 nm passes through a slit of width $2.00\,\mu\text{m}$ and produces a diffraction pattern similar to that shown in **Figure 4.9**. (a) Find the locations of the first two minima in terms of the angle from the central maximum and (b) determine the intensity relative to the central maximum at a point halfway between these two minima.

Strategy

The minima are given by **Equation 4.1**, $D\sin\theta = m\lambda$. The first two minima are for $m = 1$ and $m = 2$. **Equation 4.4** and **Equation 4.2** can be used to determine the intensity once the angle has been worked out.

Solution

a. Solving **Equation 4.1** for θ gives us $\theta_m = \sin^{-1}(m\lambda/D)$, so that

$$\theta_1 = \sin^{-1}\left(\frac{(+1)\left(550\times 10^{-9}\text{ m}\right)}{2.00\times 10^{-6}\text{ m}}\right) = +16.0°$$

and

$$\theta_2 = \sin^{-1}\left(\frac{(+2)\left(550\times 10^{-9}\text{ m}\right)}{2.00\times 10^{-6}\text{ m}}\right) = +33.4°.$$

b. The halfway point between θ_1 and θ_2 is

$$\theta = (\theta_1 + \theta_2)/2 = (16.0° + 33.4°)/2 = 24.7°.$$

Equation 4.2 gives

$$\beta = \frac{\pi D\sin\theta}{\lambda} = \frac{\pi\left(2.00\times 10^{-6}\text{ m}\right)\sin(24.7°)}{\left(550\times 10^{-9}\text{ m}\right)} = 1.52\pi \text{ or } 4.77 \text{ rad}.$$

From **Equation 4.4**, we can calculate

$$\frac{I}{I_o} = \left(\frac{\sin\beta}{\beta}\right)^2 = \left(\frac{\sin(4.77)}{4.77}\right)^2 = \left(\frac{-0.9985}{4.77}\right)^2 = 0.044.$$

Significance

This position, halfway between two minima, is very close to the location of the maximum, expected near $\beta = 3\pi/2$, or 1.5π.

4.2 Check Your Understanding For the experiment in **Example 4.2**, at what angle from the center is the third maximum and what is its intensity relative to the central maximum?

If the slit width D is varied, the intensity distribution changes, as illustrated in **Figure 4.10**. The central peak is distributed over the region from $\sin\theta = -\lambda/D$ to $\sin\theta = +\lambda/D$. For small θ, this corresponds to an angular width $\Delta\theta \approx 2\lambda/D$. Hence, an increase in the slit width results in a decrease in the **width of the central peak**. For a slit with $D \gg \lambda$, the central peak is very sharp, whereas if $D \approx \lambda$, it becomes quite broad.

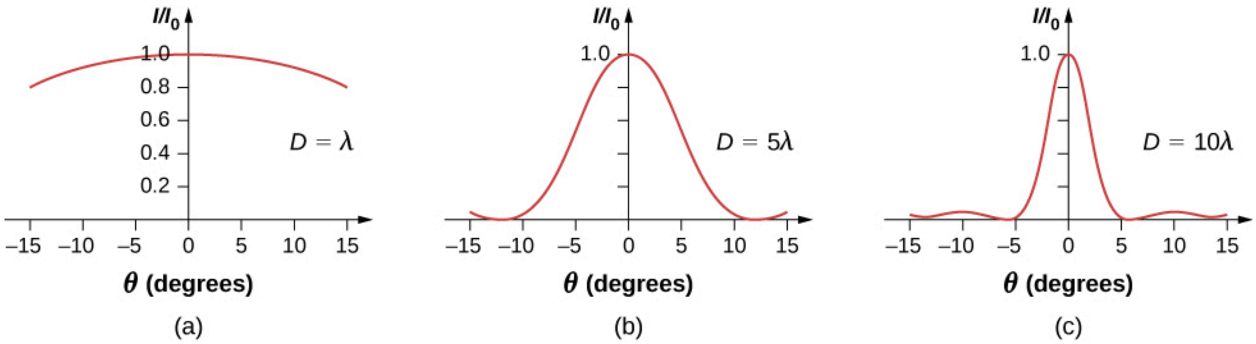


Figure 4.10 Single-slit diffraction patterns for various slit widths. As the slit width D increases from $D = \lambda$ to 5λ and then to 10λ, the width of the central peak decreases as the angles for the first minima decrease as predicted by **Equation 4.1**.

A diffraction experiment in optics can require a lot of preparation but **this simulation (https://openstaxcollege.org/l/21diffrexpoptsi)** by Andrew Duffy offers not only a quick set up but also the ability to change the slit width instantly. Run the simulation and select “Single slit.” You can adjust the slit width and see the effect on the diffraction pattern on a screen and as a graph.

4.3 | Double-Slit Diffraction

Learning Objectives

By the end of this section, you will be able to:

- Describe the combined effect of interference and diffraction with two slits, each with finite width
- Determine the relative intensities of interference fringes within a diffraction pattern
- Identify missing orders, if any

When we studied interference in Young’s double-slit experiment, we ignored the diffraction effect in each slit. We assumed that the slits were so narrow that on the screen you saw only the interference of light from just two point sources. If the slit is smaller than the wavelength, then **Figure 4.10**(a) shows that there is just a spreading of light and no peaks or troughs on the screen. Therefore, it was reasonable to leave out the diffraction effect in that chapter. However, if you make the slit wider, **Figure 4.10**(b) and (c) show that you cannot ignore diffraction. In this section, we study the complications to the double-slit experiment that arise when you also need to take into account the diffraction effect of each slit.

To calculate the diffraction pattern for two (or any number of) slits, we need to generalize the method we just used for a single slit. That is, across each slit, we place a uniform distribution of point sources that radiate Huygens wavelets, and then we sum the wavelets from all the slits. This gives the intensity at any point on the screen. Although the details of that calculation can be complicated, the final result is quite simple:

Two-Slit Diffraction Pattern

The diffraction pattern of two slits of width D that are separated by a distance d is the interference pattern of two point sources separated by d multiplied by the diffraction pattern of a slit of width D.

In other words, the *locations* of the interference fringes are given by the equation $d \sin \theta = m\lambda$, the same as when we considered the slits to be point sources, but the *intensities* of the fringes are now reduced by diffraction effects, according to **Equation 4.4**. [Note that in the chapter on interference, we wrote $d \sin \theta = m\lambda$ and used the integer m to refer to interference fringes. **Equation 4.1** also uses m, but this time to refer to diffraction minima. If both equations are used simultaneously, it is good practice to use a different variable (such as n) for one of these integers in order to keep them distinct.]

Interference and diffraction effects operate simultaneously and generally produce minima at different angles. This gives rise to a complicated pattern on the screen, in which some of the maxima of interference from the two slits are missing if the maximum of the interference is in the same direction as the minimum of the diffraction. We refer to such a missing peak as a **missing order**. One example of a diffraction pattern on the screen is shown in Figure 4.11. The solid line with multiple peaks of various heights is the intensity observed on the screen. It is a product of the interference pattern of waves from separate slits and the diffraction of waves from within one slit.

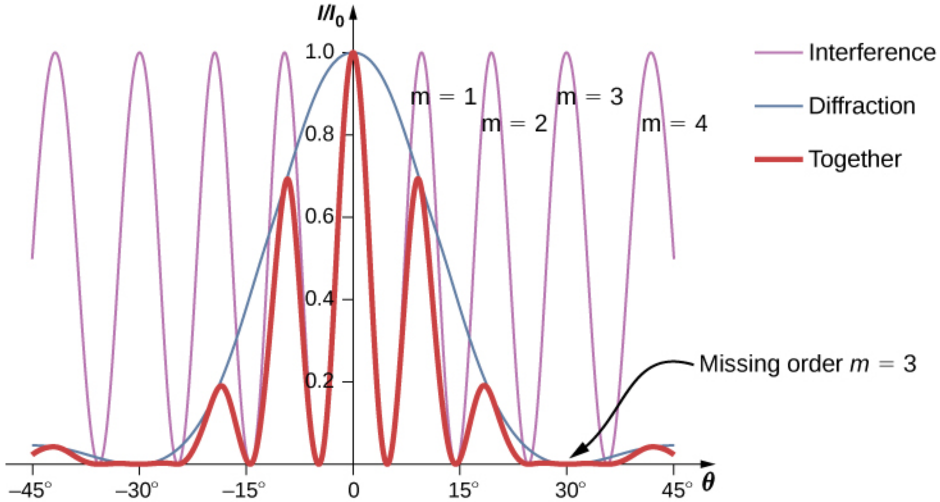


Figure 4.11 Diffraction from a double slit. The purple line with peaks of the same height are from the interference of the waves from two slits; the blue line with one big hump in the middle is the diffraction of waves from within one slit; and the thick red line is the product of the two, which is the pattern observed on the screen. The plot shows the expected result for a slit width $D = 2\lambda$ and slit separation $d = 6\lambda$. The maximum of $m = \pm 3$ order for the interference is missing because the minimum of the diffraction occurs in the same direction.

Example 4.3

Intensity of the Fringes

Figure 4.11 shows that the intensity of the fringe for $m = 3$ is zero, but what about the other fringes? Calculate the intensity for the fringe at $m = 1$ relative to I_0, the intensity of the central peak.

Strategy

Determine the angle for the double-slit interference fringe, using the equation from Interference, then determine the relative intensity in that direction due to diffraction by using Equation 4.4.

Solution

From the chapter on interference, we know that the bright interference fringes occur at $d \sin \theta = m\lambda$, or

$$\sin \theta = \frac{m\lambda}{d}.$$

From Equation 4.4,

$$I = I_0 \left(\frac{\sin \beta}{\beta}\right)^2, \text{ where } \beta = \frac{\phi}{2} = \frac{\pi D \sin \theta}{\lambda}.$$

Substituting from above,

$$\beta = \frac{\pi D \sin \theta}{\lambda} = \frac{\pi D}{\lambda} \cdot \frac{m\lambda}{d} = \frac{m\pi D}{d}.$$

For $D = 2\lambda$, $d = 6\lambda$, and $m = 1$,

$$\beta = \frac{(1)\pi(2\lambda)}{(6\lambda)} = \frac{\pi}{3}.$$

Then, the intensity is

$$I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 = I_0 \left(\frac{\sin (\pi/3)}{\pi/3} \right)^2 = 0.684 I_0.$$

Significance

Note that this approach is relatively straightforward and gives a result that is almost exactly the same as the more complicated analysis using phasors to work out the intensity values of the double-slit interference (thin line in **Figure 4.11**). The phasor approach accounts for the downward slope in the diffraction intensity (blue line) so that the peak *near* $m = 1$ occurs at a value of θ ever so slightly smaller than we have shown here.

Example 4.4

Two-Slit Diffraction

Suppose that in Young's experiment, slits of width 0.020 mm are separated by 0.20 mm. If the slits are illuminated by monochromatic light of wavelength 500 nm, how many bright fringes are observed in the central peak of the diffraction pattern?

Solution

From **Equation 4.1**, the angular position of the first diffraction minimum is $\theta \approx \sin \theta = \frac{\lambda}{D} = \frac{5.0 \times 10^{-7}\ \text{m}}{2.0 \times 10^{-5}\ \text{m}} = 2.5 \times 10^{-2}\ \text{rad}$.

Using $\sin \theta = m\lambda$ for $\theta = 2.5 \times 10^{-2}\ \text{rad}$, we find

$$m = \frac{d \sin \theta}{\lambda} = \frac{(0.20\ \text{mm})\left(2.5 \times 10^{-2}\ \text{rad}\right)}{\left(5.0 \times 10^{-7}\ \text{m}\right)} = 10,$$

which is the maximum interference order that fits inside the central peak. We note that $m = \pm 10$ are missing orders as θ matches exactly. Accordingly, we observe bright fringes for

$$m = -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, +6, +7, +8, \text{ and } +9$$

for a total of 19 bright fringes.

4.3 Check Your Understanding For the experiment in **Example 4.4**, show that $m = 20$ is also a missing order.

Explore the effects of double-slit diffraction. In **this simulation (https://openstaxcollege.org/l/21doubslitdiff)** written by Fu-Kwun Hwang, select $N = 2$ using the slider and see what happens when you control the slit width, slit separation and the wavelength. Can you make an order go "missing?"

4.4 | Diffraction Gratings

Learning Objectives

By the end of this section, you will be able to:

- Discuss the pattern obtained from diffraction gratings
- Explain diffraction grating effects

Analyzing the interference of light passing through two slits lays out the theoretical framework of interference and gives us a historical insight into Thomas Young's experiments. However, most modern-day applications of slit interference use not just two slits but many, approaching infinity for practical purposes. The key optical element is called a diffraction grating, an important tool in optical analysis.

Diffraction Gratings: An Infinite Number of Slits

The analysis of multi-slit interference in **Interference** allows us to consider what happens when the number of slits N approaches infinity. Recall that $N-2$ secondary maxima appear between the principal maxima. We can see there will be an infinite number of secondary maxima that appear, and an infinite number of dark fringes between them. This makes the spacing between the fringes, and therefore the width of the maxima, infinitesimally small. Furthermore, because the intensity of the secondary maxima is proportional to $1/N^2$, it approaches zero so that the secondary maxima are no longer seen. What remains are only the principal maxima, now very bright and very narrow (**Figure 4.12**).

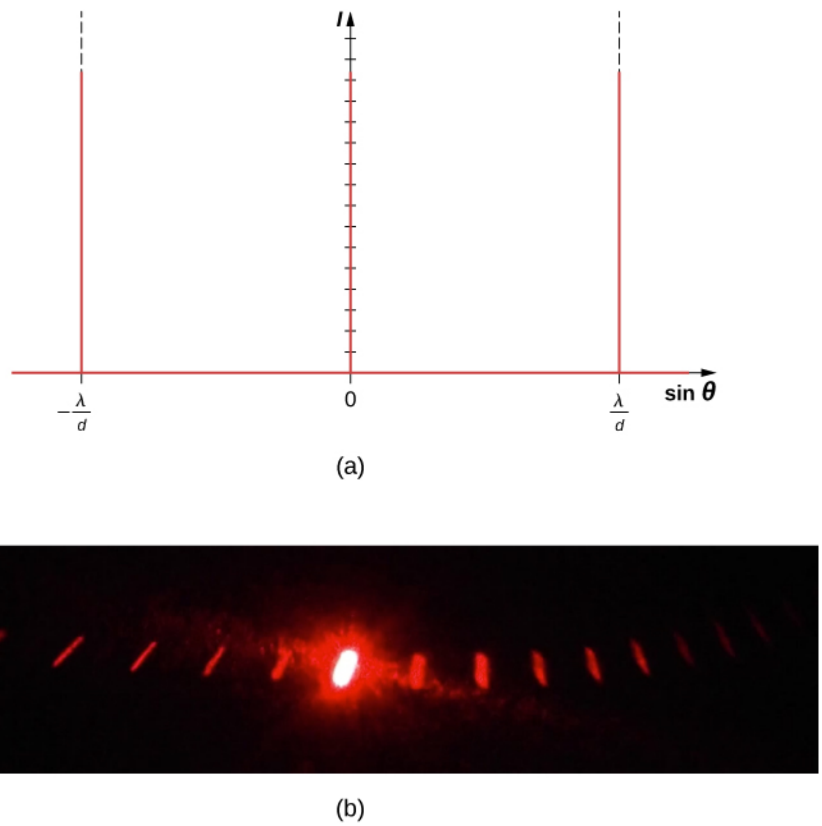


Figure 4.12 (a) Intensity of light transmitted through a large number of slits. When N approaches infinity, only the principal maxima remain as very bright and very narrow lines. (b) A laser beam passed through a diffraction grating. (credit b: modification of work by Sebastian Stapelberg)

In reality, the number of slits is not infinite, but it can be very large—large enough to produce the equivalent effect. A prime example is an optical element called a **diffraction grating**. A diffraction grating can be manufactured by carving glass with a sharp tool in a large number of precisely positioned parallel lines, with untouched regions acting like slits (**Figure 4.13**). This type of grating can be photographically mass produced rather cheaply. Because there can be over 1000 lines per millimeter across the grating, when a section as small as a few millimeters is illuminated by an incoming ray, the number of illuminated slits is effectively infinite, providing for very sharp principal maxima.

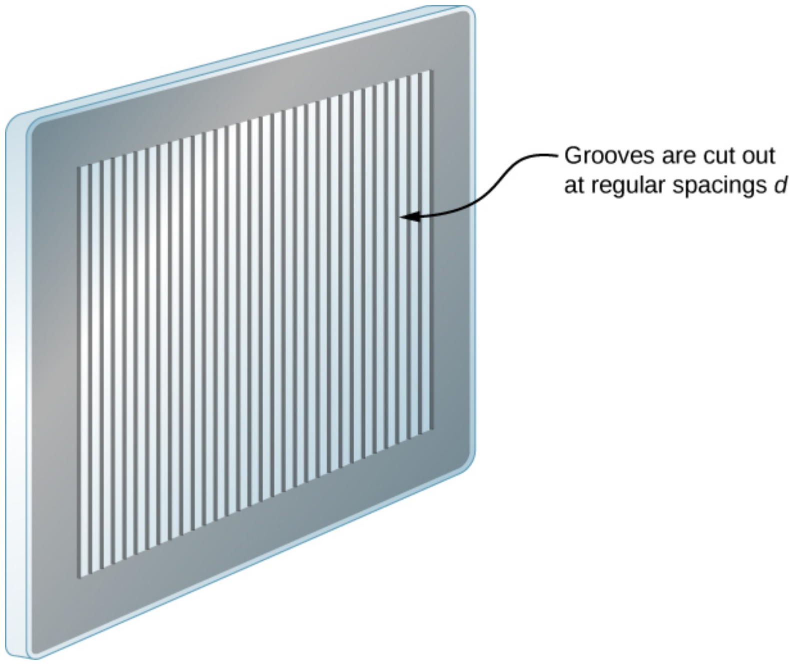


Figure 4.13 A diffraction grating can be manufactured by carving glass with a sharp tool in a large number of precisely positioned parallel lines.

Diffraction gratings work both for transmission of light, as in **Figure 4.14**, and for reflection of light, as on butterfly wings and the Australian opal in **Figure 4.15**. Natural diffraction gratings also occur in the feathers of certain birds such as the hummingbird. Tiny, finger-like structures in regular patterns act as reflection gratings, producing constructive interference that gives the feathers colors not solely due to their pigmentation. This is called iridescence.

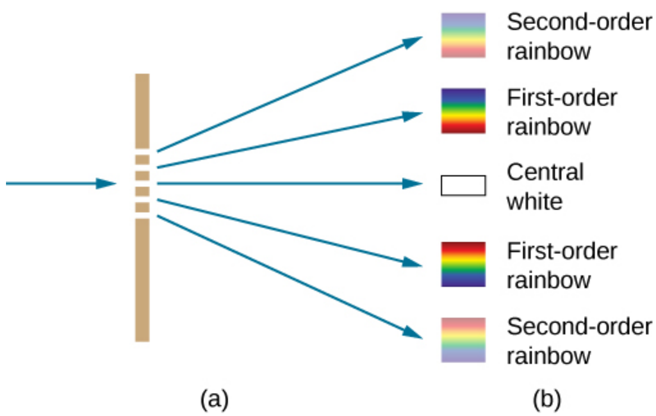


Figure 4.14 (a) Light passing through a diffraction grating is diffracted in a pattern similar to a double slit, with bright regions at various angles. (b) The pattern obtained for white light incident on a grating. The central maximum is white, and the higher-order maxima disperse white light into a rainbow of colors.

(a) (b)

Figure 4.15 (a) This Australian opal and (b) butterfly wings have rows of reflectors that act like reflection gratings, reflecting different colors at different angles. (credit a: modification of work by "Opals-On-Black"/Flickr; credit b: modification of work by “whologwhy”/Flickr)

Applications of Diffraction Gratings

Where are diffraction gratings used in applications? Diffraction gratings are commonly used for spectroscopic dispersion and analysis of light. What makes them particularly useful is the fact that they form a sharper pattern than double slits do. That is, their bright fringes are narrower and brighter while their dark regions are darker. Diffraction gratings are key components of monochromators used, for example, in optical imaging of particular wavelengths from biological or medical samples. A diffraction grating can be chosen to specifically analyze a wavelength emitted by molecules in diseased cells in a biopsy sample or to help excite strategic molecules in the sample with a selected wavelength of light. Another vital use is in optical fiber technologies where fibers are designed to provide optimum performance at specific wavelengths. A range of diffraction gratings are available for selecting wavelengths for such use.

Example 4.5

Calculating Typical Diffraction Grating Effects

Diffraction gratings with 10,000 lines per centimeter are readily available. Suppose you have one, and you send a beam of white light through it to a screen 2.00 m away. (a) Find the angles for the first-order diffraction of the shortest and longest wavelengths of visible light (380 and 760 nm, respectively). (b) What is the distance between the ends of the rainbow of visible light produced on the screen for first-order interference? (See **Figure 4.16**.)

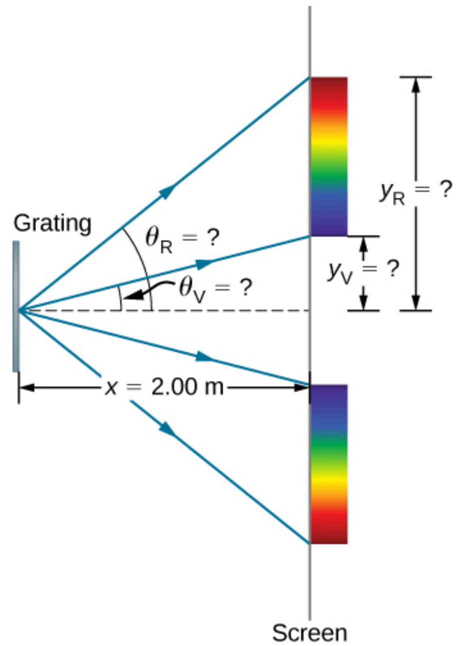


Figure 4.16 (a) The diffraction grating considered in this example produces a rainbow of colors on a screen a distance $x = 2.00\text{ m}$ from the grating. The distances along the screen are measured perpendicular to the *x*-direction. In other words, the rainbow pattern extends out of the page.
(b) In a bird's-eye view, the rainbow pattern can be seen on a table where the equipment is placed.

Strategy

Once a value for the diffraction grating's slit spacing *d* has been determined, the angles for the sharp lines can be found using the equation

$$d \sin \theta = m\lambda \text{ for } m = 0, \ \pm 1, \ \pm 2, \ \ldots .$$

Since there are 10,000 lines per centimeter, each line is separated by 1/10,000 of a centimeter. Once we know the angles, we an find the distances along the screen by using simple trigonometry.

Solution

a. The distance between slits is $d = (1\text{ cm})/10,000 = 1.00 \times 10^{-4}$ cm or 1.00×10^{-6} m. Let us call the two angles θ_V for violet (380 nm) and θ_R for red (760 nm). Solving the equation $d \sin \theta_V = m\lambda$ for $\sin \theta_V$,

$$\sin \theta_V = \frac{m\lambda_V}{d},$$

where $m = 1$ for the first-order and $\lambda_V = 380\text{ nm} = 3.80 \times 10^{-7}$ m. Substituting these values gives

$$\sin \theta_V = \frac{3.80 \times 10^{-7}\text{ m}}{1.00 \times 10^{-6}\text{ m}} = 0.380.$$

Thus the angle θ_V is

$$\theta_V = \sin^{-1} 0.380 = 22.33°.$$

Similarly,

$$\sin\theta_R = \frac{7.60 \times 10^{-7}\ \text{m}}{1.00 \times 10^{-6}\ \text{m}} = 0.760.$$

Thus the angle θ_R is

$$\theta_R = \sin^{-1} 0.760 = 49.46°.$$

Notice that in both equations, we reported the results of these intermediate calculations to four significant figures to use with the calculation in part (b).

b. The distances on the secreen are labeled y_V and y_R in **Figure 4.16**. Notice that $\tan\theta = y/x$. We can solve for y_V and y_R. That is,

$$y_V = x\tan\theta_V = (2.00\ \text{m})(\tan 22.33°) = 0.815\ \text{m}$$

and

$$y_R = x\tan\theta_R = (2.00\ \text{m})(\tan 49.46°) = 2.338\ \text{m}.$$

The distance between them is therefore

$$y_R - y_V = 1.523\ \text{m}.$$

Significance

The large distance between the red and violet ends of the rainbow produced from the white light indicates the potential this diffraction grating has as a spectroscopic tool. The more it can spread out the wavelengths (greater dispersion), the more detail can be seen in a spectrum. This depends on the quality of the diffraction grating—it must be very precisely made in addition to having closely spaced lines.

4.4 Check Your Understanding If the line spacing of a diffraction grating d is not precisely known, we can use a light source with a well-determined wavelength to measure it. Suppose the first-order constructive fringe of the H_β emission line of hydrogen $(\lambda = 656.3\ \text{nm})$ is measured at $11.36°$ using a spectrometer with a diffraction grating. What is the line spacing of this grating?

Take **the same simulation (https://openstaxcollege.org/l/21doubslitdiff)** we used for double-slit diffraction and try increasing the number of slits from $N = 2$ to $N = 3, 4, 5...$. The primary peaks become sharper, and the secondary peaks become less and less pronounced. By the time you reach the maximum number of $N = 20$, the system is behaving much like a diffraction grating.

4.5 | Circular Apertures and Resolution

Learning Objectives

By the end of this section, you will be able to:

- Describe the diffraction limit on resolution
- Describe the diffraction limit on beam propagation

Light diffracts as it moves through space, bending around obstacles, interfering constructively and destructively. This can be used as a spectroscopic tool—a diffraction grating disperses light according to wavelength, for example, and is used to produce spectra—but diffraction also limits the detail we can obtain in images.

Figure 4.17(a) shows the effect of passing light through a small circular aperture. Instead of a bright spot with sharp edges, we obtain a spot with a fuzzy edge surrounded by circles of light. This pattern is caused by diffraction, similar to that produced by a single slit. Light from different parts of the circular aperture interferes constructively and destructively. The effect is most noticeable when the aperture is small, but the effect is there for large apertures as well.

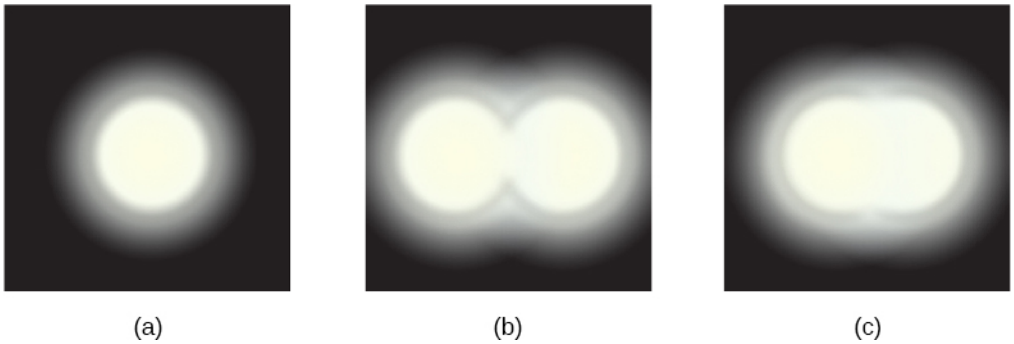


Figure 4.17 (a) Monochromatic light passed through a small circular aperture produces this diffraction pattern. (b) Two point-light sources that are close to one another produce overlapping images because of diffraction. (c) If the sources are closer together, they cannot be distinguished or resolved.

How does diffraction affect the detail that can be observed when light passes through an aperture? Figure 4.17(b) shows the diffraction pattern produced by two point-light sources that are close to one another. The pattern is similar to that for a single point source, and it is still possible to tell that there are two light sources rather than one. If they are closer together, as in Figure 4.17(c), we cannot distinguish them, thus limiting the detail or **resolution** we can obtain. This limit is an inescapable consequence of the wave nature of light.

Diffraction limits the resolution in many situations. The acuity of our vision is limited because light passes through the pupil, which is the circular aperture of the eye. Be aware that the diffraction-like spreading of light is due to the limited diameter of a light beam, not the interaction with an aperture. Thus, light passing through a lens with a diameter *D* shows this effect and spreads, blurring the image, just as light passing through an aperture of diameter *D* does. Thus, diffraction limits the resolution of any system having a lens or mirror. Telescopes are also limited by diffraction, because of the finite diameter *D* of the primary mirror.

Just what is the limit? To answer that question, consider the diffraction pattern for a circular aperture, which has a central maximum that is wider and brighter than the maxima surrounding it (similar to a slit) (Figure 4.18(a)). It can be shown that, for a circular aperture of diameter *D*, the first minimum in the diffraction pattern occurs at $\theta = 1.22\lambda/D$ (providing the aperture is large compared with the wavelength of light, which is the case for most optical instruments). The accepted criterion for determining the **diffraction limit** to resolution based on this angle is known as the **Rayleigh criterion**, which was developed by Lord Rayleigh in the nineteenth century.

Rayleigh Criterion

The diffraction limit to resolution states that two images are just resolvable when the center of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other (Figure 4.18(b)).

The first minimum is at an angle of $\theta = 1.22\lambda/D$, so that two point objects are just resolvable if they are separated by the angle

$$\theta = 1.22\frac{\lambda}{D} \tag{4.5}$$

where λ is the wavelength of light (or other electromagnetic radiation) and *D* is the diameter of the aperture, lens, mirror, etc., with which the two objects are observed. In this expression, θ has units of radians. This angle is also commonly known as the diffraction limit.

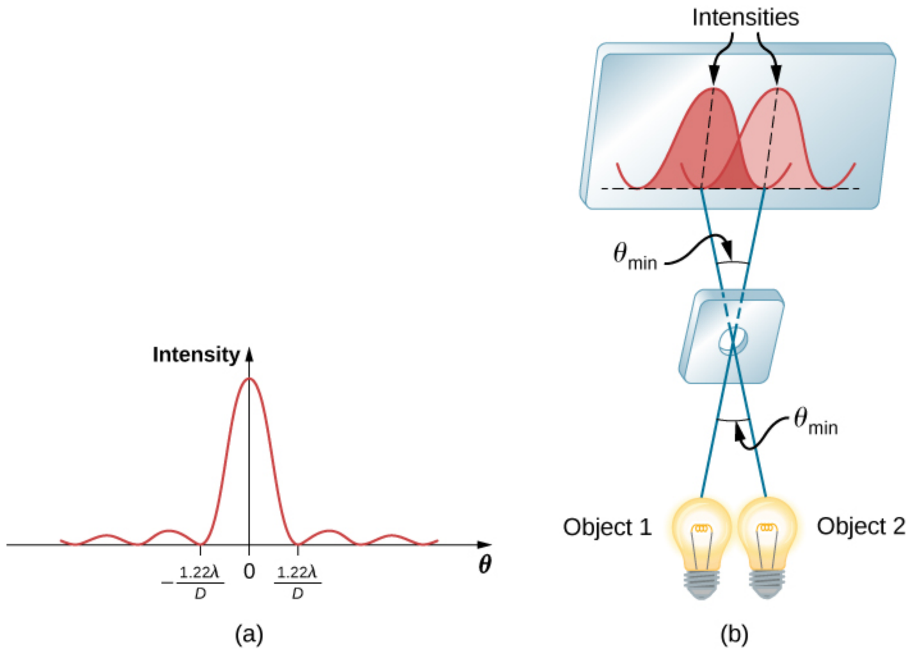


Figure 4.18 (a) Graph of intensity of the diffraction pattern for a circular aperture. Note that, similar to a single slit, the central maximum is wider and brighter than those to the sides. (b) Two point objects produce overlapping diffraction patterns. Shown here is the Rayleigh criterion for being just resolvable. The central maximum of one pattern lies on the first minimum of the other.

All attempts to observe the size and shape of objects are limited by the wavelength of the probe. Even the small wavelength of light prohibits exact precision. When extremely small wavelength probes are used, as with an electron microscope, the system is disturbed, still limiting our knowledge. Heisenberg's uncertainty principle asserts that this limit is fundamental and inescapable, as we shall see in the chapter on quantum mechanics.

Example 4.6

Calculating Diffraction Limits of the Hubble Space Telescope

The primary mirror of the orbiting Hubble Space Telescope has a diameter of 2.40 m. Being in orbit, this telescope avoids the degrading effects of atmospheric distortion on its resolution. (a) What is the angle between two just-resolvable point light sources (perhaps two stars)? Assume an average light wavelength of 550 nm. (b) If these two stars are at a distance of 2 million light-years, which is the distance of the Andromeda Galaxy, how close together can they be and still be resolved? (A light-year, or ly, is the distance light travels in 1 year.)

Strategy

The Rayleigh criterion stated in **Equation 4.5**, $\theta = 1.22\lambda/D$, gives the smallest possible angle θ between point sources, or the best obtainable resolution. Once this angle is known, we can calculate the distance between the stars, since we are given how far away they are.

Solution

a. The Rayleigh criterion for the minimum resolvable angle is

$$\theta = 1.22\frac{\lambda}{D}.$$

Entering known values gives

$$\theta = 1.22\frac{550 \times 10^{-9}\ \text{m}}{2.40\ \text{m}} = 2.80 \times 10^{-7}\ \text{rad}.$$

b. The distance s between two objects a distance r away and separated by an angle θ is $s = r\theta$.

Substituting known values gives

$$s = \left(2.0 \times 10^6 \text{ ly}\right)\left(2.80 \times 10^{-7} \text{ rad}\right) = 0.56 \text{ ly}.$$

Significance

The angle found in part (a) is extraordinarily small (less than 1/50,000 of a degree), because the primary mirror is so large compared with the wavelength of light. As noticed, diffraction effects are most noticeable when light interacts with objects having sizes on the order of the wavelength of light. However, the effect is still there, and there is a diffraction limit to what is observable. The actual resolution of the Hubble Telescope is not quite as good as that found here. As with all instruments, there are other effects, such as nonuniformities in mirrors or aberrations in lenses that further limit resolution. However, **Figure 4.19** gives an indication of the extent of the detail observable with the Hubble because of its size and quality, and especially because it is above Earth's atmosphere.

Figure 4.19 These two photographs of the M82 Galaxy give an idea of the observable detail using (a) a ground-based telescope and (b) the Hubble Space Telescope. (credit a: modification of work by "Ricnun"/Wikimedia Commons; credit b: modification of work by NASA, ESA, and The Hubble Heritage Team (STScI/AURA))

The answer in part (b) indicates that two stars separated by about half a light-year can be resolved. The average distance between stars in a galaxy is on the order of five light-years in the outer parts and about one light-year near the galactic center. Therefore, the Hubble can resolve most of the individual stars in Andromeda Galaxy, even though it lies at such a huge distance that its light takes 2 million years to reach us. **Figure 4.20** shows another mirror used to observe radio waves from outer space.

Figure 4.20 A 305-m-diameter paraboloid at Arecibo in Puerto Rico is lined with reflective material, making it into a radio telescope. It is the largest curved focusing dish in the world. Although *D* for Arecibo is much larger than for the Hubble Telescope, it detects radiation of a much longer wavelength and its diffraction limit is significantly poorer than Hubble's. The Arecibo telescope is still very useful, because important information is carried by radio waves that is not carried by visible light. (credit: Jeff Hitchcock)

4.5 Check Your Understanding What is the angular resolution of the Arecibo telescope shown in Figure 4.20 when operated at 21-cm wavelength? How does it compare to the resolution of the Hubble Telescope?

Diffraction is not only a problem for optical instruments but also for the electromagnetic radiation itself. Any beam of light having a finite diameter *D* and a wavelength λ exhibits diffraction spreading. The beam spreads out with an angle θ given by Equation 4.5, $\theta = 1.22\lambda/D$. Take, for example, a laser beam made of rays as parallel as possible (angles between rays as close to $\theta = 0°$ as possible) instead spreads out at an angle $\theta = 1.22\lambda/D$, where *D* is the diameter of the beam and λ is its wavelength. This spreading is impossible to observe for a flashlight because its beam is not very parallel to start with. However, for long-distance transmission of laser beams or microwave signals, diffraction spreading can be significant (Figure 4.21). To avoid this, we can increase *D*. This is done for laser light sent to the moon to measure its distance from Earth. The laser beam is expanded through a telescope to make *D* much larger and θ smaller.

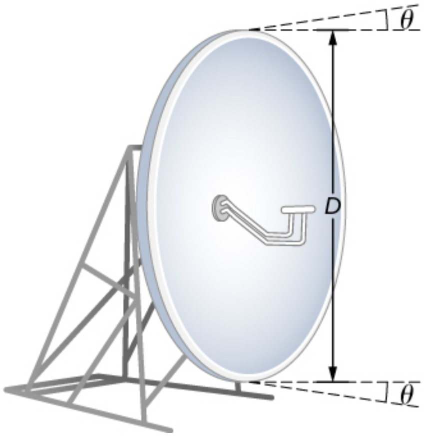


Figure 4.21 The beam produced by this microwave transmission antenna spreads out at a minimum angle $\theta = 1.22\lambda/D$ due to diffraction. It is impossible to produce a near-parallel beam because the beam has a limited diameter.

In most biology laboratories, resolution is an issue when the use of the microscope is introduced. The smaller the distance x by which two objects can be separated and still be seen as distinct, the greater the resolution. The resolving power of a lens is defined as that distance x. An expression for resolving power is obtained from the Rayleigh criterion. **Figure 4.22**(a) shows two point objects separated by a distance x. According to the Rayleigh criterion, resolution is possible when the minimum angular separation is

$$\theta = 1.22\frac{\lambda}{D} = \frac{x}{d},$$

where d is the distance between the specimen and the objective lens, and we have used the small angle approximation (i.e., we have assumed that x is much smaller than d), so that $\tan\theta \approx \sin\theta \approx \theta$. Therefore, the resolving power is

$$x = 1.22\frac{\lambda d}{D}.$$

Another way to look at this is by the concept of numerical aperture (NA), which is a measure of the maximum acceptance angle at which a lens will take light and still contain it within the lens. **Figure 4.22**(b) shows a lens and an object at point P. The NA here is a measure of the ability of the lens to gather light and resolve fine detail. The angle subtended by the lens at its focus is defined to be $\theta = 2\alpha$. From the figure and again using the small angle approximation, we can write

$$\sin\alpha = \frac{D/2}{d} = \frac{D}{2d}.$$

The NA for a lens is $NA = n\sin\alpha$, where n is the index of refraction of the medium between the objective lens and the object at point P. From this definition for NA, we can see that

$$x = 1.22\frac{\lambda d}{D} = 1.22\frac{\lambda}{2\sin\alpha} = 0.61\frac{\lambda n}{NA}.$$

In a microscope, NA is important because it relates to the resolving power of a lens. A lens with a large NA is able to resolve finer details. Lenses with larger NA are also able to collect more light and so give a brighter image. Another way to describe this situation is that the larger the NA, the larger the cone of light that can be brought into the lens, so more of the diffraction modes are collected. Thus the microscope has more information to form a clear image, and its resolving power is higher.

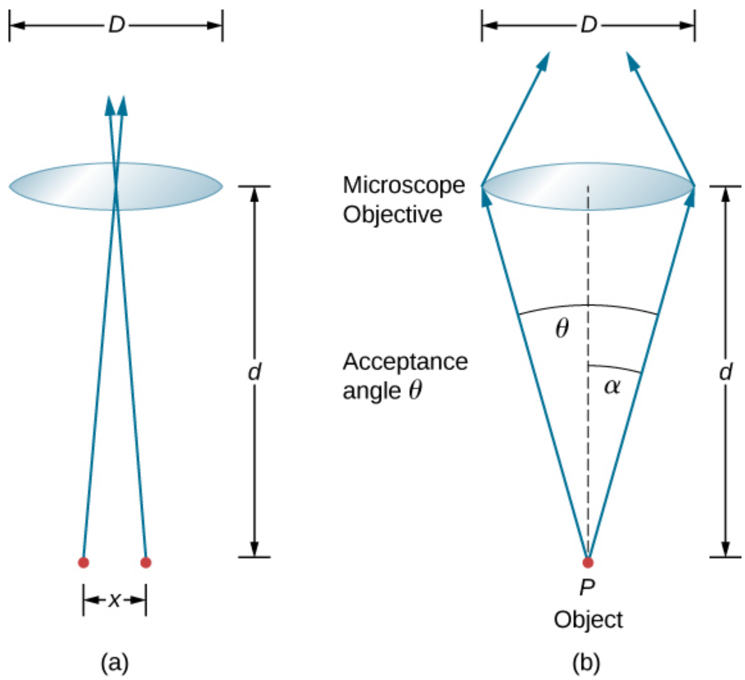


Figure 4.22 (a) Two points separated by a distance x and positioned a distance d away from the objective. (b) Terms and symbols used in discussion of resolving power for a lens and an object at point P (credit a: modification of work by “Infopro”/Wikimedia Commons).

One of the consequences of diffraction is that the focal point of a beam has a finite width and intensity distribution. Imagine focusing when only considering geometric optics, as in **Figure 4.23**(a). The focal point is regarded as an infinitely small point with a huge intensity and the capacity to incinerate most samples, irrespective of the *NA* of the objective lens—an unphysical oversimplification. For wave optics, due to diffraction, we take into account the phenomenon in which the focal point spreads to become a focal spot (**Figure 4.23**(b)) with the size of the spot decreasing with increasing *NA*. Consequently, the intensity in the focal spot increases with increasing *NA*. The higher the *NA*, the greater the chances of photodegrading the specimen. However, the spot never becomes a true point.

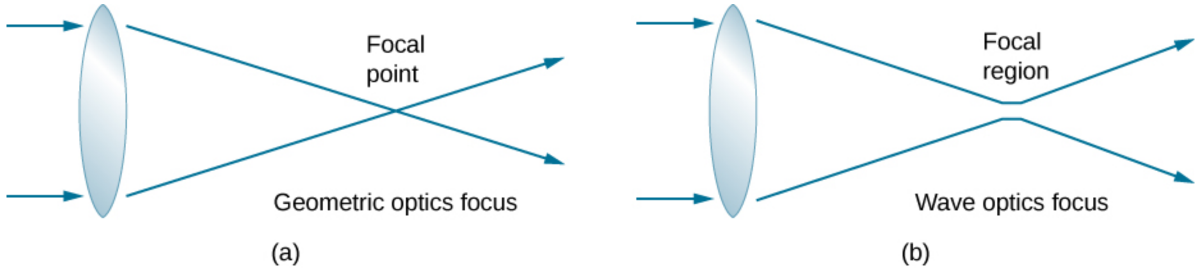


Figure 4.23 (a) In geometric optics, the focus is modelled as a point, but it is not physically possible to produce such a point because it implies infinite intensity. (b) In wave optics, the focus is an extended region.

In a different type of microscope, molecules within a specimen are made to emit light through a mechanism called fluorescence. By controlling the molecules emitting light, it has become possible to construct images with resolution much finer than the Rayleigh criterion, thus circumventing the diffraction limit. The development of super-resolved fluorescence microscopy led to the 2014 Nobel Prize in Chemistry.

In this Optical Resolution Model, two diffraction patterns for light through two circular apertures are shown side by side in **this simulation (https://openstaxcollege.org/l/21optresmodsim)** by Fu-Kwun Hwang. Watch the patterns merge as you decrease the aperture diameters.

4.6 | X-Ray Diffraction

Learning Objectives

By the end of this section, you will be able to:

- Describe interference and diffraction effects exhibited by X-rays in interaction with atomic-scale structures

Since X-ray photons are very energetic, they have relatively short wavelengths, on the order of 10^{-8} m to 10^{-12} m. Thus, typical X-ray photons act like rays when they encounter macroscopic objects, like teeth, and produce sharp shadows. However, since atoms are on the order of 0.1 nm in size, X-rays can be used to detect the location, shape, and size of atoms and molecules. The process is called **X-ray diffraction**, and it involves the interference of X-rays to produce patterns that can be analyzed for information about the structures that scattered the X-rays.

Perhaps the most famous example of X-ray diffraction is the discovery of the double-helical structure of DNA in 1953 by an international team of scientists working at England’s Cavendish Laboratory—American James Watson, Englishman Francis Crick, and New Zealand-born Maurice Wilkins. Using X-ray diffraction data produced by Rosalind Franklin, they were the first to model the double-helix structure of DNA that is so crucial to life. For this work, Watson, Crick, and Wilkins were awarded the 1962 Nobel Prize in Physiology or Medicine. (There is some debate and controversy over the issue that Rosalind Franklin was not included in the prize, although she died in 1958, before the prize was awarded.)

Figure 4.24 shows a diffraction pattern produced by the scattering of X-rays from a crystal. This process is known as X-ray crystallography because of the information it can yield about crystal structure, and it was the type of data Rosalind Franklin supplied to Watson and Crick for DNA. Not only do X-rays confirm the size and shape of atoms, they give information about the atomic arrangements in materials. For example, more recent research in high-temperature superconductors involves complex materials whose lattice arrangements are crucial to obtaining a superconducting material. These can be studied using X-ray crystallography.

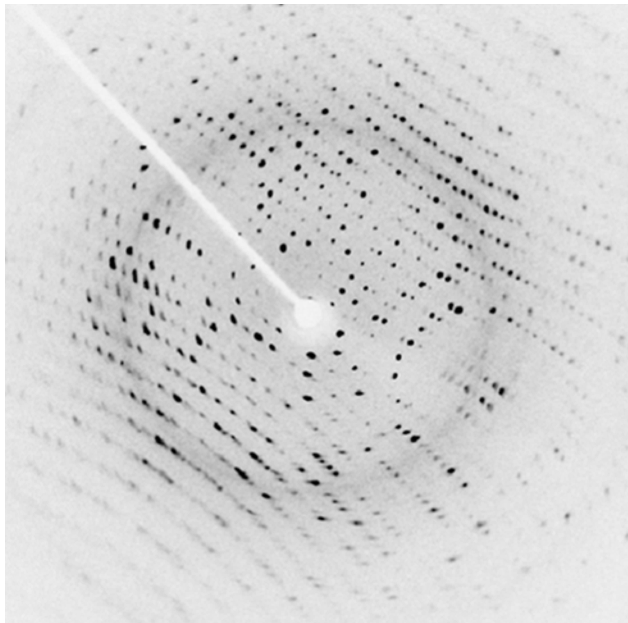


Figure 4.24 X-ray diffraction from the crystal of a protein (hen egg lysozyme) produced this interference pattern. Analysis of the pattern yields information about the structure of the protein. (credit: “Del45”/Wikimedia Commons)

Historically, the scattering of X-rays from crystals was used to prove that X-rays are energetic electromagnetic (EM) waves. This was suspected from the time of the discovery of X-rays in 1895, but it was not until 1912 that the German Max von Laue (1879–1960) convinced two of his colleagues to scatter X-rays from crystals. If a diffraction pattern is obtained, he reasoned, then the X-rays must be waves, and their wavelength could be determined. (The spacing of atoms in various crystals was reasonably well known at the time, based on good values for Avogadro’s number.) The experiments were convincing, and the 1914 Nobel Prize in Physics was given to von Laue for his suggestion leading to the proof that X-rays are EM waves. In 1915, the unique father-and-son team of Sir William Henry Bragg and his son Sir William Lawrence Bragg were awarded a joint Nobel Prize for inventing the X-ray spectrometer and the then-new science of X-ray analysis.

In ways reminiscent of thin-film interference, we consider two plane waves at X-ray wavelengths, each one reflecting off a different plane of atoms within a crystal’s lattice, as shown in **Figure 4.25**. From the geometry, the difference in path lengths is $2d \sin\theta$. Constructive interference results when this distance is an integer multiple of the wavelength. This condition is captured by the *Bragg equation*,

$$m\lambda = 2d\sin\theta,\ \ m = 1,\ \ 2,\ \ 3\ldots \tag{4.6}$$

where m is a positive integer and d is the spacing between the planes. Following the Law of Reflection, both the incident and reflected waves are described by the same angle, $\theta,$ but unlike the general practice in geometric optics, θ is measured with respect to the surface itself, rather than the normal.

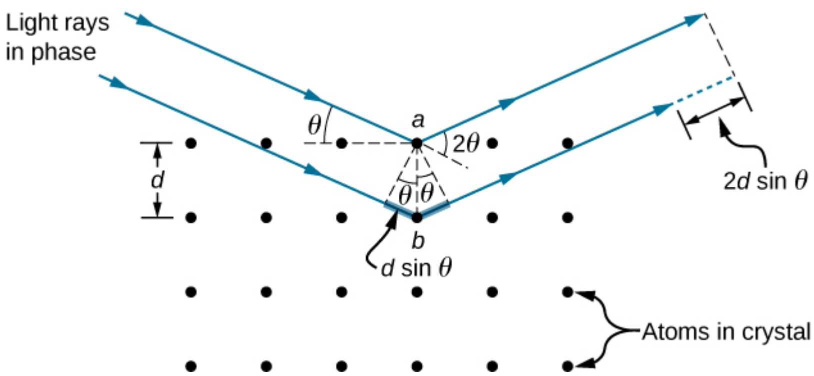


Figure 4.25 X-ray diffraction with a crystal. Two incident waves reflect off two planes of a crystal. The difference in path lengths is indicated by the dashed line.

Example 4.7

X-Ray Diffraction with Salt Crystals

Common table salt is composed mainly of NaCl crystals. In a NaCl crystal, there is a family of planes 0.252 nm apart. If the first-order maximum is observed at an incidence angle of $18.1°$, what is the wavelength of the X-ray scattering from this crystal?

Strategy

Use the Bragg equation, **Equation 4.6**, $m\lambda = 2d \sin\theta$, to solve for θ.

Solution

For first-order, $m = 1,$ and the plane spacing d is known. Solving the Bragg equation for wavelength yields

$$\lambda = \frac{2d \sin\theta}{m} = \frac{2\left(0.252 \times 10^{-9}\ \text{m}\right) \sin(18.1°)}{1} = 1.57 \times 10^{-10}\ \text{m},\ \text{ or } 0.157\ \text{nm}.$$

Significance

The determined wavelength fits within the X-ray region of the electromagnetic spectrum. Once again, the wave nature of light makes itself prominent when the wavelength $(\lambda = 0.157\ \text{nm})$ is comparable to the size of the physical structures $(d = 0.252\ \text{nm})$ it interacts with.

4.6 Check Your Understanding For the experiment described in **Example 4.7**, what are the two other angles where interference maxima may be observed? What limits the number of maxima?

Although **Figure 4.25** depicts a crystal as a two-dimensional array of scattering centers for simplicity, real crystals are structures in three dimensions. Scattering can occur simultaneously from different families of planes at different orientations and spacing patterns known as called **Bragg planes**, as shown in **Figure 4.26**. The resulting interference pattern can be quite complex.

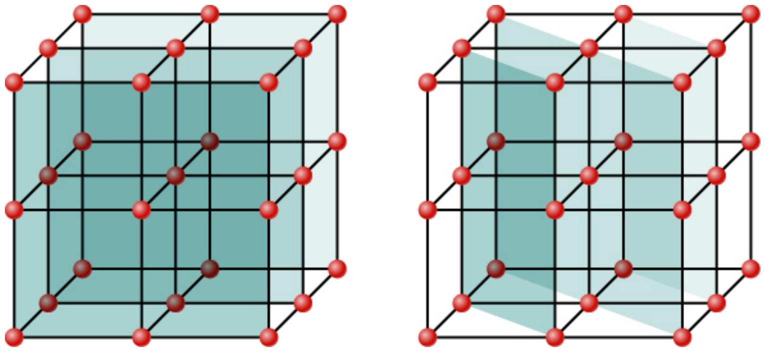


Figure 4.26 Because of the regularity that makes a crystal structure, one crystal can have many families of planes within its geometry, each one giving rise to X-ray diffraction.

4.7 | Holography

Learning Objectives

By the end of this section, you will be able to:

- Describe how a three-dimensional image is recorded as a hologram
- Describe how a three-dimensional image is formed from a hologram

A **hologram**, such as the one in Figure 4.27, is a true three-dimensional image recorded on film by lasers. Holograms are used for amusement; decoration on novelty items and magazine covers; security on credit cards and driver's licenses (a laser and other equipment are needed to reproduce them); and for serious three-dimensional information storage. You can see that a hologram is a true three-dimensional image because objects change relative position in the image when viewed from different angles.

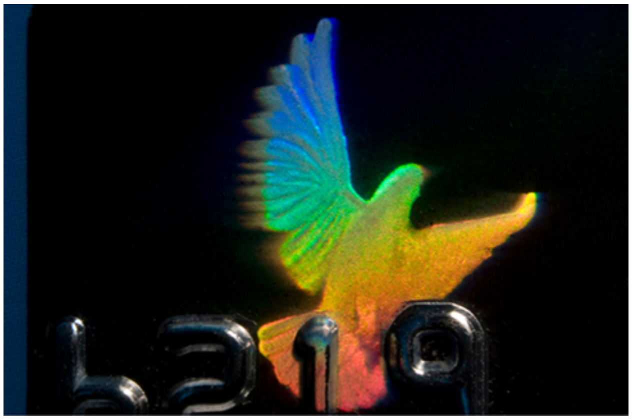


Figure 4.27 Credit cards commonly have holograms for logos, making them difficult to reproduce. (credit: Dominic Alves)

The name hologram means "entire picture" (from the Greek *holo*, as in holistic) because the image is three-dimensional. **Holography** is the process of producing holograms and, although they are recorded on photographic film, the process is quite different from normal photography. Holography uses light interference or wave optics, whereas normal photography uses geometric optics. Figure 4.28 shows one method of producing a hologram. Coherent light from a laser is split by a mirror, with part of the light illuminating the object. The remainder, called the reference beam, shines directly on a piece of film. Light scattered from the object interferes with the reference beam, producing constructive and destructive interference. As a result, the exposed film looks foggy, but close examination reveals a complicated interference pattern stored on it. Where the interference was constructive, the film (a negative actually) is darkened. Holography is sometimes called lens-less photography, because it uses the wave characteristics of light, as contrasted to normal photography, which uses geometric optics and requires lenses.

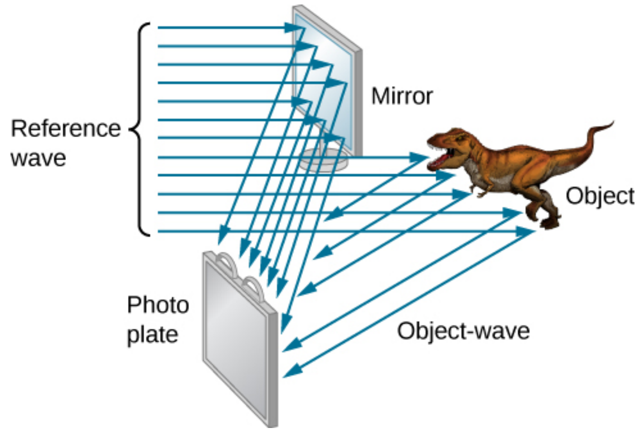


Figure 4.28 Production of a hologram. Single-wavelength coherent light from a laser produces a well-defined interference pattern on a piece of film. The laser beam is split by a partially silvered mirror, with part of the light illuminating the object and the remainder shining directly on the film. (credit: modification of work by Mariana Ruiz Villarreal)

Light falling on a hologram can form a three-dimensional image of the original object. The process is complicated in detail, but the basics can be understood, as shown in **Figure 4.29**, in which a laser of the same type that exposed the film is now used to illuminate it. The myriad tiny exposed regions of the film are dark and block the light, whereas less exposed regions allow light to pass. The film thus acts much like a collection of diffraction gratings with various spacing patterns. Light passing through the hologram is diffracted in various directions, producing both real and virtual images of the object used to expose the film. The interference pattern is the same as that produced by the object. Moving your eye to various places in the interference pattern gives you different perspectives, just as looking directly at the object would. The image thus looks like the object and is three dimensional like the object.

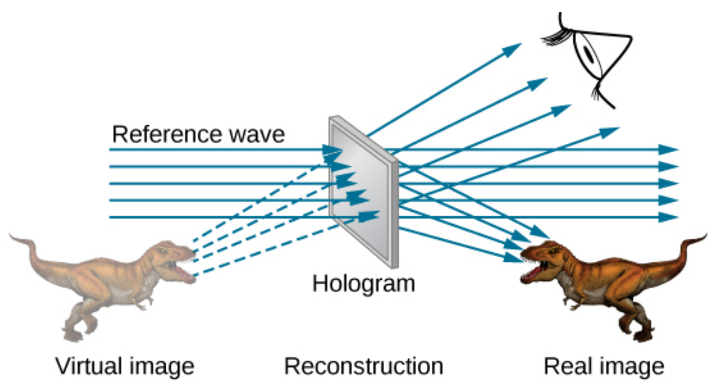


Figure 4.29 A transmission hologram is one that produces real and virtual images when a laser of the same type as that which exposed the hologram is passed through it. Diffraction from various parts of the film produces the same interference pattern that was produced by the object that was used to expose it. (credit: modification of work by Mariana Ruiz Villarreal)

The hologram illustrated in **Figure 4.29** is a transmission hologram. Holograms that are viewed with reflected light, such as the white light holograms on credit cards, are reflection holograms and are more common. White light holograms often appear a little blurry with rainbow edges, because the diffraction patterns of various colors of light are at slightly different locations due to their different wavelengths. Further uses of holography include all types of three-dimensional information storage, such as of statues in museums, engineering studies of structures, and images of human organs.

Invented in the late 1940s by Dennis Gabor (1900–1970), who won the 1971 Nobel Prize in Physics for his work, holography became far more practical with the development of the laser. Since lasers produce coherent single-wavelength light, their interference patterns are more pronounced. The precision is so great that it is even possible to record numerous holograms on a single piece of film by just changing the angle of the film for each successive image. This is how the holograms that move as you walk by them are produced—a kind of lens-less movie.

In a similar way, in the medical field, holograms have allowed complete three-dimensional holographic displays of objects from a stack of images. Storing these images for future use is relatively easy. With the use of an endoscope, high-resolution, three-dimensional holographic images of internal organs and tissues can be made.

CHAPTER 4 REVIEW

KEY TERMS

Bragg planes families of planes within crystals that can give rise to X-ray diffraction

destructive interference for a single slit occurs when the width of the slit is comparable to the wavelength of light illuminating it

diffraction bending of a wave around the edges of an opening or an obstacle

diffraction grating large number of evenly spaced parallel slits

diffraction limit fundamental limit to resolution due to diffraction

hologram three-dimensional image recorded on film by lasers; the word hologram means *entire picture* (from the Greek word *holo*, as in holistic)

holography process of producing holograms with the use of lasers

missing order interference maximum that is not seen because it coincides with a diffraction minimum

Rayleigh criterion two images are just-resolvable when the center of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other

resolution ability, or limit thereof, to distinguish small details in images

two-slit diffraction pattern diffraction pattern of two slits of width D that are separated by a distance d is the interference pattern of two point sources separated by d multiplied by the diffraction pattern of a slit of width D

width of the central peak angle between the minimum for $m = 1$ and $m = -1$

X-ray diffraction technique that provides the detailed information about crystallographic structure of natural and manufactured materials

KEY EQUATIONS

Destructive interference for a single slit	$D \sin\theta = m\lambda$ for $m = \pm 1, \pm 2, \pm 3, ...$
Half phase angle	$\beta = \frac{\phi}{2} = \frac{\pi D \sin\theta}{\lambda}$
Field amplitude in the diffraction pattern	$E = N\Delta E_0 \frac{\sin\beta}{\beta}$
Intensity in the diffraction pattern	$I = I_0 \left(\frac{\sin\beta}{\beta}\right)^2$
Rayleigh criterion for circular apertures	$\theta = 1.22\frac{\lambda}{D}$
Bragg equation	$m\lambda = 2d \sin\theta,\ m = 1, 2, 3...$

SUMMARY

4.1 Single-Slit Diffraction

- Diffraction can send a wave around the edges of an opening or other obstacle.
- A single slit produces an interference pattern characterized by a broad central maximum with narrower and dimmer maxima to the sides.

4.2 Intensity in Single-Slit Diffraction

- The intensity pattern for diffraction due to a single slit can be calculated using phasors as

$$I = I_0\left(\frac{\sin\beta}{\beta}\right)^2,$$

where $\beta = \frac{\phi}{2} = \frac{\pi D \sin\theta}{\lambda}$, D is the slit width, λ is the wavelength, and θ is the angle from the central peak.

4.3 Double-Slit Diffraction

- With real slits with finite widths, the effects of interference and diffraction operate simultaneously to form a complicated intensity pattern.
- Relative intensities of interference fringes within a diffraction pattern can be determined.
- Missing orders occur when an interference maximum and a diffraction minimum are located together.

4.4 Diffraction Gratings

- A diffraction grating consists of a large number of evenly spaced parallel slits that produce an interference pattern similar to but sharper than that of a double slit.
- Constructive interference occurs when $d\sin\theta = m\lambda$ for $m = 0, \pm 1, \pm 2, ...,$ where d is the distance between the slits, θ is the angle relative to the incident direction, and m is the order of the interference.

4.5 Circular Apertures and Resolution

- Diffraction limits resolution.
- The Rayleigh criterion states that two images are just resolvable when the center of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other.

4.6 X-Ray Diffraction

- X-rays are relatively short-wavelength EM radiation and can exhibit wave characteristics such as interference when interacting with correspondingly small objects.

4.7 Holography

- Holography is a technique based on wave interference to record and form three-dimensional images.
- Lasers offer a practical way to produce sharp holographic images because of their monochromatic and coherent light for pronounced interference patterns.

CONCEPTUAL QUESTIONS

4.1 Single-Slit Diffraction

1. As the width of the slit producing a single-slit diffraction pattern is reduced, how will the diffraction pattern produced change?

2. Compare interference and diffraction.

3. If you and a friend are on opposite sides of a hill, you can communicate with walkie-talkies but not with flashlights. Explain.

4. What happens to the diffraction pattern of a single slit when the entire optical apparatus is immersed in water?

5. In our study of diffraction by a single slit, we assume that the length of the slit is much larger than the width. What happens to the diffraction pattern if these two dimensions were comparable?

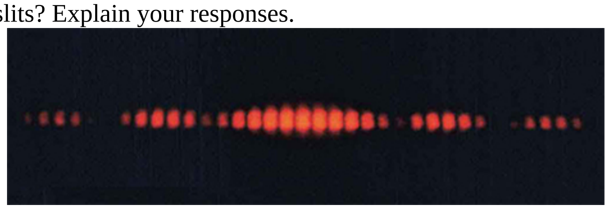
6. A rectangular slit is twice as wide as it is high. Is the central diffraction peak wider in the vertical direction or in the horizontal direction?

4.2 Intensity in Single-Slit Diffraction

7. In **Equation 4.4**, the parameter β looks like an angle but is not an angle that you can measure with a protractor in the physical world. Explain what β represents.

4.3 Double-Slit Diffraction

8. Shown below is the central part of the interference pattern for a pure wavelength of red light projected onto a double slit. The pattern is actually a combination of single- and double-slit interference. Note that the bright spots are evenly spaced. Is this a double- or single-slit characteristic? Note that some of the bright spots are dim on either side of the center. Is this a single- or double-slit characteristic? Which is smaller, the slit width or the separation between slits? Explain your responses.



(credit: PASCO)

4.5 Circular Apertures and Resolution

9. Is higher resolution obtained in a microscope with red or blue light? Explain your answer.

10. The resolving power of refracting telescope increases with the size of its objective lens. What other advantage is gained with a larger lens?

11. The distance between atoms in a molecule is about 10^{-8} cm. Can visible light be used to "see" molecules?

12. A beam of light always spreads out. Why can a beam not be created with parallel rays to prevent spreading? Why can lenses, mirrors, or apertures not be used to correct the spreading?

4.6 X-Ray Diffraction

13. Crystal lattices can be examined with X-rays but not UV. Why?

4.7 Holography

14. How can you tell that a hologram is a true three-dimensional image and that those in three-dimensional movies are not?

15. If a hologram is recorded using monochromatic light at one wavelength but its image is viewed at another wavelength, say 10% shorter, what will you see? What if it is viewed using light of exactly half the original wavelength?

16. What image will one see if a hologram is recorded using monochromatic light but its image is viewed in white light? Explain.

PROBLEMS

4.1 Single-Slit Diffraction

17. (a) At what angle is the first minimum for 550-nm light falling on a single slit of width $1.00 \mu m$? (b) Will there be a second minimum?

18. (a) Calculate the angle at which a $2.00\text{-}\mu m$-wide slit produces its first minimum for 410-nm violet light. (b) Where is the first minimum for 700-nm red light?

19. (a) How wide is a single slit that produces its first minimum for 633-nm light at an angle of $28.0°$? (b) At what angle will the second minimum be?

20. (a) What is the width of a single slit that produces its first minimum at $60.0°$ for 600-nm light? (b) Find the wavelength of light that has its first minimum at $62.0°$.

21. Find the wavelength of light that has its third minimum at an angle of $48.6°$ when it falls on a single slit of width $3.00 \mu m$.

22. (a) Sodium vapor light averaging 589 nm in wavelength falls on a single slit of width $7.50 \mu m$. At what angle does it produces its second minimum? (b) What is the highest-order minimum produced?

23. Consider a single-slit diffraction pattern for $\lambda = 589$ nm, projected on a screen that is 1.00 m from a slit of width 0.25 mm. How far from the center of the pattern are the centers of the first and second dark fringes?

24. (a) Find the angle between the first minima for the two sodium vapor lines, which have wavelengths of 589.1 and 589.6 nm, when they fall upon a single slit of width $2.00 \mu m$. (b) What is the distance between these minima if the diffraction pattern falls on a screen 1.00 m from the slit? (c) Discuss the ease or difficulty of measuring such a distance.

25. (a) What is the minimum width of a single slit (in multiples of λ) that will produce a first minimum for a wavelength λ ? (b) What is its minimum width if it produces 50 minima? (c) 1000 minima?

26. (a) If a single slit produces a first minimum at $14.5°$, at what angle is the second-order minimum? (b) What is the angle of the third-order minimum? (c) Is there a fourth-order minimum? (d) Use your answers to illustrate how the angular width of the central maximum is about twice the angular width of the next maximum (which is the angle between the first and second minima).

27. If the separation between the first and the second minima of a single-slit diffraction pattern is 6.0 mm, what is the distance between the screen and the slit? The light wavelength is 500 nm and the slit width is 0.16 mm.

28. A water break at the entrance to a harbor consists of a rock barrier with a 50.0-m-wide opening. Ocean waves of 20.0-m wavelength approach the opening straight on. At what angles to the incident direction are the boats inside the harbor most protected against wave action?

29. An aircraft maintenance technician walks past a tall hangar door that acts like a single slit for sound entering the hangar. Outside the door, on a line perpendicular to the opening in the door, a jet engine makes a 600-Hz sound. At what angle with the door will the technician observe the first minimum in sound intensity if the vertical opening is 0.800 m wide and the speed of sound is 340 m/s?

4.2 Intensity in Single-Slit Diffraction

30. A single slit of width $3.0\,\mu\text{m}$ is illuminated by a sodium yellow light of wavelength 589 nm. Find the intensity at a $15°$ angle to the axis in terms of the intensity of the central maximum.

31. A single slit of width 0.1 mm is illuminated by a mercury light of wavelength 576 nm. Find the intensity at a $10°$ angle to the axis in terms of the intensity of the central maximum.

32. The width of the central peak in a single-slit diffraction pattern is 5.0 mm. The wavelength of the light is 600 nm, and the screen is 2.0 m from the slit. (a) What is the width of the slit? (b) Determine the ratio of the intensity at 4.5 mm from the center of the pattern to the intensity at the center.

33. Consider the single-slit diffraction pattern for $\lambda = 600\text{ nm}$, $D = 0.025\text{ mm}$, and $x = 2.0\text{ m}$. Find the intensity in terms of I_o at $\theta = 0.5°$, $1.0°$, $1.5°$, $3.0°$, and $10.0°$.

4.3 Double-Slit Diffraction

34. Two slits of width $2\,\mu\text{m}$, each in an opaque material, are separated by a center-to-center distance of $6\,\mu\text{m}$. A monochromatic light of wavelength 450 nm is incident on the double-slit. One finds a combined interference and diffraction pattern on the screen.

(a) How many peaks of the interference will be observed in the central maximum of the diffraction pattern?

(b) How many peaks of the interference will be observed if the slit width is doubled while keeping the distance between the slits same?

(c) How many peaks of interference will be observed if the slits are separated by twice the distance, that is, $12\,\mu\text{m}$, while keeping the widths of the slits same?

(d) What will happen in (a) if instead of 450-nm light another light of wavelength 680 nm is used?

(e) What is the value of the ratio of the intensity of the central peak to the intensity of the next bright peak in (a)?

(f) Does this ratio depend on the wavelength of the light?

(g) Does this ratio depend on the width or separation of the slits?

35. A double slit produces a diffraction pattern that is a combination of single- and double-slit interference. Find the ratio of the width of the slits to the separation between them, if the first minimum of the single-slit pattern falls on the fifth maximum of the double-slit pattern. (This will greatly reduce the intensity of the fifth maximum.)

36. For a double-slit configuration where the slit separation is four times the slit width, how many interference fringes lie in the central peak of the diffraction pattern?

37. Light of wavelength 500 nm falls normally on 50 slits that are 2.5×10^{-3} mm wide and spaced 5.0×10^{-3} mm apart. How many interference fringes lie in the central peak of the diffraction pattern?

38. A monochromatic light of wavelength 589 nm incident on a double slit with slit width $2.5\,\mu\text{m}$ and unknown separation results in a diffraction pattern containing nine interference peaks inside the central maximum. Find the separation of the slits.

39. When a monochromatic light of wavelength 430 nm incident on a double slit of slit separation $5\ \mu m$, there are 11 interference fringes in its central maximum. How many interference fringes will be in the central maximum of a light of wavelength 632.8 nm for the same double slit?

40. Determine the intensities of two interference peaks other than the central peak in the central maximum of the diffraction, if possible, when a light of wavelength 628 nm is incident on a double slit of width 500 nm and separation 1500 nm. Use the intensity of the central spot to be $1\ mW/cm^2$.

4.4 Diffraction Gratings

41. A diffraction grating has 2000 lines per centimeter. At what angle will the first-order maximum be for 520-nm-wavelength green light?

42. Find the angle for the third-order maximum for 580-nm-wavelength yellow light falling on a difraction grating having 1500 lines per centimeter.

43. How many lines per centimeter are there on a diffraction grating that gives a first-order maximum for 470-nm blue light at an angle of $25.0°$?

44. What is the distance between lines on a diffraction grating that produces a second-order maximum for 760-nm red light at an angle of $60.0°$?

45. Calculate the wavelength of light that has its second-order maximum at $45.0°$ when falling on a diffraction grating that has 5000 lines per centimeter.

46. An electric current through hydrogen gas produces several distinct wavelengths of visible light. What are the wavelengths of the hydrogen spectrum, if they form first-order maxima at angles $24.2°$, $25.7°$, $29.1°$, and $41.0°$ when projected on a diffraction grating having 10,000 lines per centimeter?

47. (a) What do the four angles in the preceding problem become if a 5000-line per centimeter diffraction grating is used? (b) Using this grating, what would the angles be for the second-order maxima? (c) Discuss the relationship between integral reductions in lines per centimeter and the new angles of various order maxima.

48. What is the spacing between structures in a feather that acts as a reflection grating, giving that they produce a first-order maximum for 525-nm light at a $30.0°$ angle?

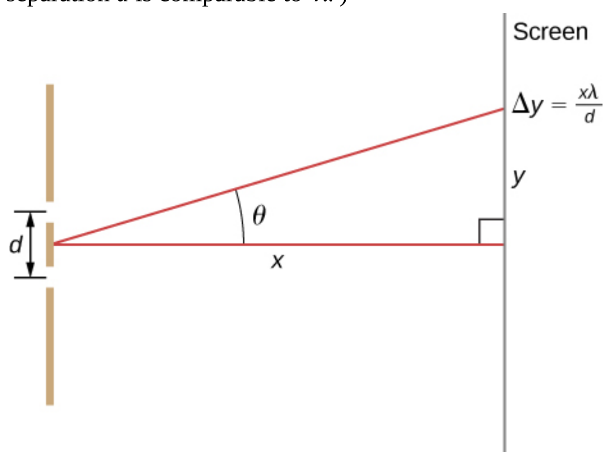
49. An opal such as that shown in **Figure 4.15** acts like a reflection grating with rows separated by about $8\ \mu m$. If the opal is illuminated normally, (a) at what angle will red light be seen and (b) at what angle will blue light be seen?

50. At what angle does a diffraction grating produce a second-order maximum for light having a first-order maximum at $20.0°$?

51. (a) Find the maximum number of lines per centimeter a diffraction grating can have and produce a maximum for the smallest wavelength of visible light. (b) Would such a grating be useful for ultraviolet spectra? (c) For infrared spectra?

52. (a) Show that a 30,000 line per centimeter grating will not produce a maximum for visible light. (b) What is the longest wavelength for which it does produce a first-order maximum? (c) What is the greatest number of line per centimeter a diffraction grating can have and produce a complete second-order spectrum for visible light?

53. The analysis shown below also applies to diffraction gratings with lines separated by a distance *d*. What is the distance between fringes produced by a diffraction grating having 125 lines per centimeter for 600-nm light, if the screen is 1.50 m away? (*Hint:* The distance between adjacent fringes is $\Delta y = x\lambda/d$, assuming the slit separation *d* is comparable to λ.)



4.5 Circular Apertures and Resolution

54. The 305-m-diameter Arecibo radio telescope pictured in **Figure 4.20** detects radio waves with a 4.00-cm average wavelength. (a) What is the angle between two just-resolvable point sources for this telescope? (b) How close together could these point sources be at the 2 million light-year distance of the Andromeda Galaxy?

55. Assuming the angular resolution found for the Hubble Telescope in **Example 4.6**, what is the smallest detail that could be observed on the moon?

56. Diffraction spreading for a flashlight is insignificant compared with other limitations in its optics, such as spherical aberrations in its mirror. To show this, calculate the minimum angular spreading of a flashlight beam that is originally 5.00 cm in diameter with an average wavelength of 600 nm.

57. (a) What is the minimum angular spread of a 633-nm wavelength He-Ne laser beam that is originally 1.00 mm in diameter? (b) If this laser is aimed at a mountain cliff 15.0 km away, how big will the illuminated spot be? (c) How big a spot would be illuminated on the moon, neglecting atmospheric effects? (This might be done to hit a corner reflector to measure the round-trip time and, hence, distance.)

58. A telescope can be used to enlarge the diameter of a laser beam and limit diffraction spreading. The laser beam is sent through the telescope in opposite the normal direction and can then be projected onto a satellite or the moon. (a) If this is done with the Mount Wilson telescope, producing a 2.54-m-diameter beam of 633-nm light, what is the minimum angular spread of the beam? (b) Neglecting atmospheric effects, what is the size of the spot this beam would make on the moon, assuming a lunar distance of 3.84×10^8 m ?

59. The limit to the eye's acuity is actually related to diffraction by the pupil. (a) What is the angle between two just-resolvable points of light for a 3.00-mm-diameter pupil, assuming an average wavelength of 550 nm? (b) Take your result to be the practical limit for the eye. What is the greatest possible distance a car can be from you if you can resolve its two headlights, given they are 1.30 m apart? (c) What is the distance between two just-resolvable points held at an arm's length (0.800 m) from your eye? (d) How does your answer to (c) compare to details you normally observe in everyday circumstances?

60. What is the minimum diameter mirror on a telescope that would allow you to see details as small as 5.00 km on the moon some 384,000 km away? Assume an average wavelength of 550 nm for the light received.

61. Find the radius of a star's image on the retina of an eye if its pupil is open to 0.65 cm and the distance from the pupil to the retina is 2.8 cm. Assume $\lambda = 550\ \text{nm}$.

62. (a) The dwarf planet Pluto and its moon, Charon, are separated by 19,600 km. Neglecting atmospheric effects, should the 5.08-m-diameter Palomar Mountain telescope be able to resolve these bodies when they are 4.50×10^9 km from Earth? Assume an average wavelength of 550 nm. (b) In actuality, it is just barely possible to discern that Pluto and Charon are separate bodies using a ground-based telescope. What are the reasons for this?

63. A spy satellite orbits Earth at a height of 180 km. What is the minimum diameter of the objective lens in a telescope that must be used to resolve columns of troops marching 2.0 m apart? Assume $\lambda = 550\ \text{nm}$.

64. What is the minimum angular separation of two stars that are just-resolvable by the 8.1-m Gemini South telescope, if atmospheric effects do not limit resolution? Use 550 nm for the wavelength of the light from the stars.

65. The headlights of a car are 1.3 m apart. What is the maximum distance at which the eye can resolve these two headlights? Take the pupil diameter to be 0.40 cm.

66. When dots are placed on a page from a laser printer, they must be close enough so that you do not see the individual dots of ink. To do this, the separation of the dots must be less than Raleigh's criterion. Take the pupil of the eye to be 3.0 mm and the distance from the paper to the eye of 35 cm; find the minimum separation of two dots such that they cannot be resolved. How many dots per inch (dpi) does this correspond to?

67. Suppose you are looking down at a highway from a jetliner flying at an altitude of 6.0 km. How far apart must two cars be if you are able to distinguish them? Assume that $\lambda = 550\ \text{nm}$ and that the diameter of your pupils is 4.0 mm.

68. Can an astronaut orbiting Earth in a satellite at a distance of 180 km from the surface distinguish two skyscrapers that are 20 m apart? Assume that the pupils of the astronaut's eyes have a diameter of 5.0 mm and that most of the light is centered around 500 nm.

69. The characters of a stadium scoreboard are formed with closely spaced lightbulbs that radiate primarily yellow light. (Use $\lambda = 600\ \text{nm}.$) How closely must the bulbs be spaced so that an observer 80 m away sees a display of continuous lines rather than the individual bulbs? Assume that the pupil of the observer's eye has a diameter of 5.0 mm.

70. If a microscope can accept light from objects at angles as large as $\alpha = 70°$, what is the smallest structure that can be resolved when illuminated with light of wavelength 500 nm and (a) the specimen is in air? (b) When the specimen is immersed in oil, with index of refraction of 1.52?

71. A camera uses a lens with aperture 2.0 cm. What is the angular resolution of a photograph taken at 700 nm wavelength? Can it resolve the millimeter markings of a ruler placed 35 m away?

4.6 X-Ray Diffraction

72. X-rays of wavelength 0.103 nm reflects off a crystal and a second-order maximum is recorded at a Bragg angle of $25.5°$. What is the spacing between the scattering planes in this crystal?

73. A first-order Bragg reflection maximum is observed when a monochromatic X-ray falls on a crystal at a $32.3°$ angle to a reflecting plane. What is the wavelength of this X-ray?

74. An X-ray scattering experiment is performed on a crystal whose atoms form planes separated by 0.440 nm. Using an X-ray source of wavelength 0.548 nm, what is the angle (with respect to the planes in question) at which the experimenter needs to illuminate the crystal in order to observe a first-order maximum?

75. The structure of the NaCl crystal forms reflecting planes 0.541 nm apart. What is the smallest angle, measured from these planes, at which X-ray diffraction can be observed, if X-rays of wavelength 0.085 nm are used?

76. On a certain crystal, a first-order X-ray diffraction maximum is observed at an angle of $27.1°$ relative to its surface, using an X-ray source of unknown wavelength. Additionally, when illuminated with a different, this time of known wavelength 0.137 nm, a second-order maximum is detected at $37.3°$. Determine (a) the spacing between the reflecting planes, and (b) the unknown wavelength.

77. Calcite crystals contain scattering planes separated by 0.30 nm. What is the angular separation between first and second-order diffraction maxima when X-rays of 0.130 nm wavelength are used?

78. The first-order Bragg angle for a certain crystal is $12.1°$. What is the second-order angle?

ADDITIONAL PROBLEMS

79. White light falls on two narrow slits separated by 0.40 mm. The interference pattern is observed on a screen 3.0 m away. (a) What is the separation between the first maxima for red light $(\lambda = 700\,\text{nm})$ and violet light $(\lambda = 400\,\text{nm})$? (b) At what point nearest the central maximum will a maximum for yellow light $(\lambda = 600\,\text{nm})$ coincide with a maximum for violet light? Identify the order for each maximum.

80. Microwaves of wavelength 10.0 mm fall normally on a metal plate that contains a slit 25 mm wide. (a) Where are the first minima of the diffraction pattern? (b) Would there be minima if the wavelength were 30.0 mm?

81. *Quasars*, or *quasi-stellar radio sources*, are astronomical objects discovered in 1960. They are distant but strong emitters of radio waves with angular size so small, they were originally unresolved, the same as stars. The quasar 3C405 is actually two discrete radio sources that subtend an angle of 82 arcsec. If this object is studied using radio emissions at a frequency of 410 MHz, what is the minimum diameter of a radio telescope that can resolve the two sources?

82. Two slits each of width 1800 nm and separated by the center-to-center distance of 1200 nm are illuminated by plane waves from a krypton ion laser-emitting at wavelength 461.9 nm. Find the number of interference peaks in the central diffraction peak.

83. A microwave of an unknown wavelength is incident on a single slit of width 6 cm. The angular width of the central peak is found to be $25°$. Find the wavelength.

84. Red light (wavelength 632.8 nm in air) from a Helium-Neon laser is incident on a single slit of width 0.05 mm. The entire apparatus is immersed in water of refractive index 1.333. Determine the angular width of the central peak.

85. A light ray of wavelength 461.9 nm emerges from a 2-mm circular aperture of a krypton ion laser. Due to diffraction, the beam expands as it moves out. How large is the central bright spot at (a) 1 m, (b) 1 km, (c) 1000 km, and (d) at the surface of the moon at a distance of 400,000 km from Earth.

86. How far apart must two objects be on the moon to be distinguishable by eye if only the diffraction effects of the eye’s pupil limit the resolution? Assume 550 nm for the wavelength of light, the pupil diameter 5.0 mm, and 400,000 km for the distance to the moon.

87. How far apart must two objects be on the moon to be resolvable by the 8.1-m-diameter Gemini North telescope at Mauna Kea, Hawaii, if only the diffraction effects of the telescope aperture limit the resolution? Assume 550 nm for the wavelength of light and 400,000 km for the distance to the moon.

88. A spy satellite is reputed to be able to resolve objects 10. cm apart while operating 197 km above the surface of Earth. What is the diameter of the aperture of the telescope if the resolution is only limited by the diffraction effects? Use 550 nm for light.

89. Monochromatic light of wavelength 530 nm passes through a horizontal single slit of width $1.5\,\mu\text{m}$ in an opaque plate. A screen of dimensions $2.0\,\text{m} \times 2.0\,\text{m}$ is 1.2 m away from the slit. (a) Which way is the diffraction pattern spread out on the screen? (b) What are the angles of the minima with respect to the center? (c) What are the angles of the maxima? (d) How wide is the central bright fringe on the screen? (e) How wide is the next bright fringe on the screen?

90. A monochromatic light of unknown wavelength is incident on a slit of width $20\,\mu\text{m}$. A diffraction pattern is seen at a screen 2.5 m away where the central maximum is spread over a distance of 10.0 cm. Find the wavelength.

91. A source of light having two wavelengths 550 nm and 600 nm of equal intensity is incident on a slit of width $1.8\,\mu\text{m}$. Find the separation of the $m = 1$ bright spots of the two wavelengths on a screen 30.0 cm away.

92. A single slit of width 2100 nm is illuminated normally by a wave of wavelength 632.8 nm. Find the phase difference between waves from the top and one third from the bottom of the slit to a point on a screen at a horizontal distance of 2.0 m and vertical distance of 10.0 cm from the center.

93. A single slit of width $3.0\,\mu\text{m}$ is illuminated by a sodium yellow light of wavelength 589 nm. Find the intensity at a $15°$ angle to the axis in terms of the intensity of the central maximum.

94. A single slit of width 0.10 mm is illuminated by a mercury lamp of wavelength 576 nm. Find the intensity at a $10°$ angle to the axis in terms of the intensity of the central maximum.

95. A diffraction grating produces a second maximum that is 89.7 cm from the central maximum on a screen 2.0 m away. If the grating has 600 lines per centimeter, what is the wavelength of the light that produces the diffraction pattern?

96. A grating with 4000 lines per centimeter is used to diffract light that contains all wavelengths between 400 and 650 nm. How wide is the first-order spectrum on a screen 3.0 m from the grating?

97. A diffraction grating with 2000 lines per centimeter is used to measure the wavelengths emitted by a hydrogen gas discharge tube. (a) At what angles will you find the maxima of the two first-order blue lines of wavelengths 410 and 434 nm? (b) The maxima of two other first-order lines are found at $\theta_1 = 0.097\,\text{rad}$ and $\theta_2 = 0.132\,\text{rad}$. What are the wavelengths of these lines?

98. For white light $(400\,\text{nm} < \lambda < 700\,\text{nm})$ falling normally on a diffraction grating, show that the second and third-order spectra overlap no matter what the grating constant *d* is.

99. How many complete orders of the visible spectrum $(400\,\text{nm} < \lambda < 700\,\text{nm})$ can be produced with a diffraction grating that contains 5000 lines per centimeter?

100. Two lamps producing light of wavelength 589 nm are fixed 1.0 m apart on a wooden plank. What is the maximum distance an observer can be and still resolve the lamps as two separate sources of light, if the resolution is affected solely by the diffraction of light entering the eye? Assume light enters the eye through a pupil of diameter 4.5 mm.

101. On a bright clear day, you are at the top of a mountain and looking at a city 12 km away. There are two tall towers 20.0 m apart in the city. Can your eye resolve the two towers if the diameter of the pupil is 4.0 mm? If not, what should be the minimum magnification power of the telescope needed to resolve the two towers? In your calculations use 550 nm for the wavelength of the light.

102. Radio telescopes are telescopes used for the detection of radio emission from space. Because radio waves have much longer wavelengths than visible light, the diameter of a radio telescope must be very large to provide good resolution. For example, the radio telescope in Penticton, BC in Canada, has a diameter of 26 m and can be operated at frequencies as high as 6.6 GHz. (a) What is the wavelength corresponding to this frequency? (b) What is the angular separation of two radio sources that can be resolved by this telescope? (c) Compare the telescope's resolution with the angular size of the moon.

Figure 4.30 (credit: modification of work by Jason Nishiyama)

103. Calculate the wavelength of light that produces its first minimum at an angle of $36.9°$ when falling on a single slit of width $1.00\ \mu\text{m}$.

104. (a) Find the angle of the third diffraction minimum for 633-nm light falling on a slit of width $20.0\ \mu\text{m}$. (b) What slit width would place this minimum at $85.0°$?

105. As an example of diffraction by apertures of everyday dimensions, consider a doorway of width 1.0 m. (a) What is the angular position of the first minimum in the diffraction pattern of 600-nm light? (b) Repeat this calculation for a musical note of frequency 440 Hz (A above middle C). Take the speed of sound to be 343 m/s.

106. What are the angular positions of the first and second minima in a diffraction pattern produced by a slit of width 0.20 mm that is illuminated by 400 nm light? What is the angular width of the central peak?

107. How far would you place a screen from the slit of the previous problem so that the second minimum is a distance of 2.5 mm from the center of the diffraction pattern?

108. How narrow is a slit that produces a diffraction pattern on a screen 1.8 m away whose central peak is 1.0 m wide? Assume $\lambda = 589\ \text{nm}$.

109. Suppose that the central peak of a single-slit diffraction pattern is so wide that the first minima can be assumed to occur at angular positions of $\pm 90°$. For this case, what is the ratio of the slit width to the wavelength of the light?

110. The central diffraction peak of the double-slit interference pattern contains exactly nine fringes. What is the ratio of the slit separation to the slit width?

111. Determine the intensities of three interference peaks other than the central peak in the central maximum of the diffraction, if possible, when a light of wavelength 500 nm is incident normally on a double slit of width 1000 nm and separation 1500 nm. Use the intensity of the central spot to be $1\ \text{mW/cm}^2$.

112. The yellow light from a sodium vapor lamp *seems* to be of pure wavelength, but it produces two first-order maxima at $36.093°$ and $36.129°$ when projected on a 10,000 line per centimeter diffraction grating. What are the two wavelengths to an accuracy of 0.1 nm?

113. Structures on a bird feather act like a reflection grating having 8000 lines per centimeter. What is the angle of the first-order maximum for 600-nm light?

114. If a diffraction grating produces a first-order maximum for the shortest wavelength of visible light at $30.0°$, at what angle will the first-order maximum be for the largest wavelength of visible light?

115. (a) What visible wavelength has its fourth-order maximum at an angle of $25.0°$ when projected on a 25,000-line per centimeter diffraction grating? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

116. Consider a spectrometer based on a diffraction grating. Construct a problem in which you calculate the distance between two wavelengths of electromagnetic radiation in your spectrometer. Among the things to be considered are the wavelengths you wish to be able to distinguish, the number of lines per meter on the diffraction grating, and the distance from the grating to the screen or detector. Discuss the practicality of the device in terms of being able to discern between wavelengths of interest.

117. An amateur astronomer wants to build a telescope with a diffraction limit that will allow him to see if there are people on the moons of Jupiter. (a) What diameter mirror is needed to be able to see 1.00-m detail on a Jovian moon at a distance of $7.50 \times 10^8\ \text{km}$ from Earth? The wavelength of light averages 600 nm. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

CHALLENGE PROBLEMS

118. Blue light of wavelength 450 nm falls on a slit of width 0.25 mm. A converging lens of focal length 20 cm is placed behind the slit and focuses the diffraction pattern on a screen. (a) How far is the screen from the lens? (b) What is the distance between the first and the third minima of the diffraction pattern?

119. (a) Assume that the maxima are halfway between the minima of a single-slit diffraction pattern. The use the diameter and circumference of the phasor diagram, as described in **Intensity in Single-Slit Diffraction**, to determine the intensities of the third and fourth maxima in terms of the intensity of the central maximum. (b) Do the same calculation, using **Equation 4.4**.

120. (a) By differentiating **Equation 4.4**, show that the higher-order maxima of the single-slit diffraction pattern occur at values of β that satisfy $\tan\beta = \beta$. (b) Plot $y = \tan\beta$ and $y = \beta$ versus β and find the intersections of these two curves. What information do they give you about the locations of the maxima? (c) Convince yourself that these points do not appear exactly at $\beta = \left(n + \frac{1}{2}\right)\pi$, where $n = 0,\ 1,\ 2,\ \ldots,$ but are quite close to these values.

121. What is the maximum number of lines per centimeter a diffraction grating can have and produce a complete first-order spectrum for visible light?

122. Show that a diffraction grating cannot produce a second-order maximum for a given wavelength of light unless the first-order maximum is at an angle less than $30.0°$.

123. A He-Ne laser beam is reflected from the surface of a CD onto a wall. The brightest spot is the reflected beam at an angle equal to the angle of incidence. However, fringes are also observed. If the wall is 1.50 m from the CD, and the first fringe is 0.600 m from the central maximum, what is the spacing of grooves on the CD?

124. Objects viewed through a microscope are placed very close to the focal point of the objective lens. Show that the minimum separation x of two objects resolvable through the microscope is given by $x = \frac{1.22\lambda f_0}{D}$, where f_0 is the focal length and D is the diameter of the objective lens as shown below.